University of São Paulo

São Carlos Institute of Physics

Graduate Program

Admission Test

Computational Physics

First Semester 2026

Exam Booklet

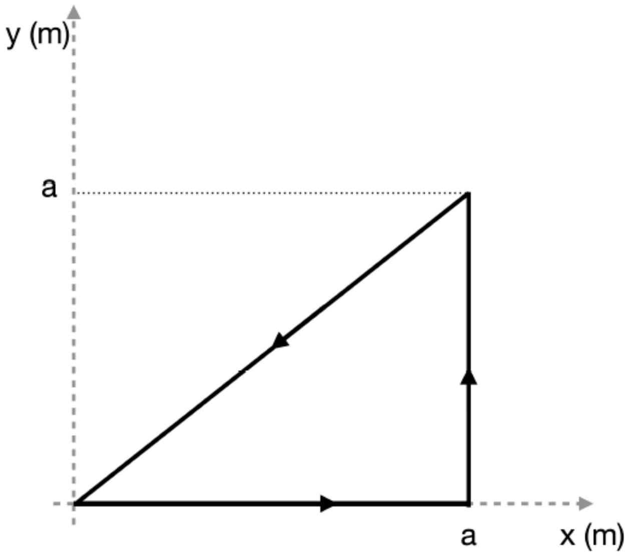
Candidate's Code:

Physics Questions
(Multiple Choice)

Instructions: The Physics questions are multiple-choice. For these questions, please indicate your chosen answer directly in this exam booklet by marking the corresponding square with an "X" using a black or blue pen. Do not use this exam booklet for elaborating on your answers or as a draft. Additionally, you will receive a separate notepaper for drafting and calculations. **The answers considered for correction will be the ones marked in the exam booklet.**

Question 1:

A particle moves along a closed circuit in the xy-plane shown in the figure, starting its motion at the origin, under the action of a force $\mathbf{F_1}=A(y^3\mathbf{i}-2xy^2\mathbf{j})$, with A a constant in N/m³. In this circuit, the force $\mathbf{F_1}$ does work W_1 on the particle. Another particle follows the same circuit under the action of a different force $\mathbf{F_2}=(aA)(x^2\mathbf{i}-y^2\mathbf{j})$, which does work W_2 along this path. We are interested in the difference $W_1 - W_2$.



Place an 'X' in the square corresponding to the correct answer.

☐ $W_1 - W_2 = (4/3)Aa^4$

☐ $W_1 - W_2 = (1/2)Aa^4$

☒ $W_1 - W_2 = (-5/12)Aa^4$

☐ $W_1 - W_2 = 0$

☐ $W_1 - W_2 = (-4/3)Aa^4$

Answer:

The work done by the force $\vec{F}_2$ in the closed circuit shown in the figure, and in any closed circuit, is zero because it is a conservative field. This can be seen since

$$\frac{\partial F_{2,x}}{\partial y} = \frac{\partial F_{2,y}}{\partial x}.$$

The work done by $\vec{F}_1$ can be calculated in 3 parts. In the first, from the origin to $(a, 0)$, the work is zero because the force is zero. In the second segment, up to (a, a), with $\vec{d\ell}_2 = dy\,\hat{\jmath}$, the work is

$$W^{(2)}_{F_1} = \int \vec{F}_1 \cdot d\vec{\ell}_2 = -2aA \int_0^a dy\, y^2 = -\frac{2}{3}Aa^4.$$

In the third and final segment we have $x = y$ and $dx = dy$, and

$$W^{(3)}_{F_1} = \int \vec{F}_1 \cdot d\vec{\ell}_3 = -A \int_a^0 dx\, x^3 = \frac{A}{4}a^4.$$

Adding the three segments we have

$$W_{\text{Tot}} = -\frac{5}{12}Aa^4.$$

Thus, the final answer is

$$W_1 - W_2 = -\frac{5}{12}Aa^4.$$

Question 2:

A block of mass M, on a horizontal surface, is pulled by a force of constant magnitude $F = (3/5)Mg$, making an angle θ with the horizontal axis. Suppose the force is sufficient to set the block in motion without lifting it off the ground. There is kinetic friction and the coefficient of friction is $\mu = 1/\sqrt{3}$. Assuming the force is applied at the angle that maximizes the acceleration, what is the time t the block takes to travel a distance $d = (\sqrt{3})$ m? [Use $g = 10\ m/s^2$.]

Place an 'X' in the square that corresponds to the correct answer.

□ $t=3s$

X $t=(\sqrt{3})s$

□ $t=2(\sqrt{3})s$

□ $t=(1/\sqrt{3})s$

□ $t=3(\sqrt{3})s$

Answer:

The acceleration of the block is

$$a = \frac{1}{M}[F\cos\theta - (Mg - F\sin\theta)\mu] = \frac{F}{M}(\cos\theta + \mu\sin\theta) - g\mu.$$

The angle θ_0 that maximizes the acceleration is therefore

$$\theta = \arctan(\mu),$$

and since $\mu = \frac{1}{\sqrt{3}}$, then $\theta_0 = 30°$. Under the conditions given in the problem, the acceleration is

$$a = \frac{1}{5\sqrt{3}}g.$$

The time the block takes to travel the distance $d = \sqrt{3}\,\mathrm{m}$ is given by

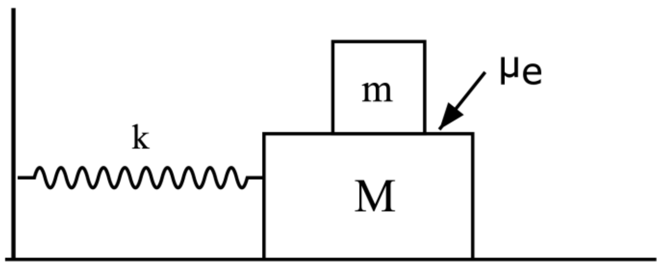
$$\frac{1}{2}at^2 = d,$$

and finally

$$t = \sqrt{3}\,\mathrm{s}.$$

Question 3:

A block of mass $M = 2\,kg$ slides on a frictionless horizontal surface and is connected to a spring with spring constant $k = 200\,N/m$ and negligible mass, which is fixed to a vertical wall. A smaller block of mass $m = 1\,kg$ whose surface has a static friction coefficient $\mu_e = 0.4$ is placed on top of the larger block, as shown in the figure. What is the maximum amplitude, in centimeters, of the simple harmonic motion of the block–spring system so that the smaller block does not slip on the larger block? If necessary, use $g = 10\,m/s^2$.



Place an 'X' in the square that corresponds to the correct answer.

☐ 3

☐ 4

☒ 6

☐ 12

☐ None of the previous answers

Answer:

Under the condition that there is no slipping, the two blocks perform a simple harmonic motion with total mass $M_T = M + m$. Thus, the angular frequency of the oscillation is:

$$\omega^2 = \frac{k}{(M+m)}$$

For the upper block not to slip, it is necessary that the friction force be the net force acting on the upper block, leading to the acceleration of the harmonic motion.

We are interested in the extreme case, of maximum amplitude, and the friction force arises from static friction. The maximum value of the acceleration of the harmonic motion, in magnitude, is

$$a_{max} = \omega^2 A$$

where A is the amplitude of the oscillation. Using that the net force on the upper block must be zero, the force due to a_max must be compensated by the static friction force. Using again the extreme case, in which $\mathrm{f}_{\mathrm{at}} = \mu_e m g$,

$$a_{max} = \omega^2 A$$

Solving for A:

$$A = \frac{\mu_e g}{\omega^2}$$

we find:

$$A = \frac{\mu_e g(M+m)}{k}$$

With the values given, we find $A = 6cm$.

Question 4:

A 15cm long violin string, fixed at both ends, vibrates in its fundamental mode (n=1). The speed of waves on the string is 250 m/s and the speed of sound in air is 348 m/s. Consider that the speed of waves on the string is equal to the square root of tension (τ) divided by the string linear density (μ). What are the frequency (ν) and the wavelength (λ) of the emitted sound wave in air and, the tension on the string (τ), if the linear string density is 0.65 g/m?

Place an 'X' in the square that corresponds to the correct answer.

X ν=833 Hz, λ=0.418 m and τ=40.6 N

☐ ν=1667 Hz, λ=0.209 m and τ=40.6 N

☐ ν=833 Hz, λ=0.209 m and τ=40.6 N

☐ ν=833 Hz, λ=0.418 m and τ=78.7 N

☐ None of the previous answers

Answer:

Frequency:

$\nu = v/\lambda = 250\ (m/s) / 0.3\ m = 833\ Hz$

Wavelength of the sound wave:

$\lambda = v/\nu = 348\ (m/s) / 833\ Hz = 0.418\ m$

Tension in the string:

$v^2 = \tau/\mu$

$\tau = v^2\ \mu = (250\ m/s)^2 \times 0.65 \times 10^{-3}\ (kg/m) = 40.6\ N$

Question 5:

One mole of a monoatomic ideal gas is heated from 300 to 600 K by two distinct quasi-static processes. First, at constant volume and, second, at constant pressure. Determine the heat added (Q), the work done by the gas (W) and the change in the internal energy (ΔU) for these two processes. If necessary, use that the universal gas constant is R=8.31 J/(mol.K).

Place an 'X' in the square that corresponds to the correct answer.

☒ First: Q=3.74 kJ, W=0 J and ΔU=3.74 kJ. Second: Q=6.23 kJ, W=2.49 kJ and ΔU=3.74 kJ.

☐ First: Q=3.74 kJ, W=0 J and ΔU=3.74 kJ. Second: Q=3.73 kJ, W=8.73 kJ and ΔU=3.74 kJ.

☐ First: Q=6.24 kJ, W=3.74 J and ΔU=6.24 kJ. Second: Q=6.23 kJ, W=2.49 kJ and ΔU=3.74 kJ.

☐ First: Q=3.74 kJ, W=0 J and ΔU=3.74 kJ. Second: Q=3.73 kJ, W=3.74 kJ and ΔU=3.74 kJ.

☐ None of the previous answers

Answer:

First, at constant volume:

Work done by the gas: $W = 0$ J

From the first law of thermodynamics: $\Delta U = Q - W = Q$

$\Delta U = Q = C_v \Delta T = (3/2) nR \Delta T = 1.5 \times 1 \times 8.31 \times (600 - 300) = 3.74$ kJ

Second, at constant pressure:

Work done by the gas: $W = P \Delta V$

From the ideal gas equation: $P \Delta V = nR \Delta T \rightarrow W = nR \Delta T = 1 \times 8.31 \times (600 - 300) = 2.49$ kJ

Heat supplied at constant pressure: $Q = C_p \Delta T = (C_v + nR) \Delta T = (5/2) nR \Delta T = 2.5 \times 1 \times 8.31 \times (600 - 300) = 6.23$ kJ

From the first law of thermodynamics: $\Delta U = Q - W = 6.23 - 2.49 = 3.74$ kJ

Computing Science Questions
(Open Answers)

Instructions: All questions related to computer science are in the form of open-ended questions. Therefore, you are expected to provide a comprehensive response for each question in the designated space within this examination booklet. Please use a black or blue pen for your responses. Additionally, you will receive a separate notepaper for drafting and calculations, but **only the answers provided within the allocated space in the exam booklet will be considered for evaluation.**

Question 1:

A positive integer n is said to be perfect if it equals the sum of all its proper divisors. An integer m is a proper divisor of n if and only if $1 \leq m < n$ and the remainder of the division of n by m is zero.

For instance, 6 is perfect because $1 + 2 + 3 = 6$. Also, 28 is perfect because $1 + 2 + 4 + 7 + 14 = 28$. But 12 is not perfect, because $1 + 2 + 3 + 4 + 6 = 16$.

Write a function called `is_perfect` that takes an integer as input and returns a boolean true value if the number is perfect, and false otherwise.

Use C, C++, Fortran or Java in your code.

Space to answer Question 1 (page 1):

```
bool is_perfect(int n) {
  int sum_divisors = 0;
  for (int i = 1; i <= n / 2; ++i) {
    if (n % i == 0) {
      sum_divisors += i;
    }
  }
  return (sum_divisors == n);
}
```

Question 2:

Write a function called `merge` that takes two sorted lists of integer values as input and returns a new list containing all the elements from the original lists in sorted order. The original lists may have different lengths.

For example, given the inputs [1, 4, 10] and [3, 4, 7, 15], the function should return [1, 3, 4, 4, 7, 10, 15].

Use C, C++, Fortran or Java in your code.

Space to answer Question 2 (page 1):

```
int *merge(int *v1, size_t n1, int *v2, size_t n2) {
  int *result = (int *)malloc((n1 + n2) * sizeof(int));

  size_t i1 = 0, i2 = 0, ir = 0;
  while (i1 < n1 && i2 < n2) {
    result[ir++] = (v1[i1] <= v2[i2]) ? v1[i1++] : v2[i2++];
  }
  while (i1 < n1)
    result[ir++] = v1[i1++];
  while (i2 < n2)
    result[ir++] = v2[i2++];

  return result;
}
```

Question 3:

Describe the bisection method used for finding a root of a given function $f(x)$. Include a pseudocode of the method in your answer.

Space to answer Question 3 (page 1):

The bisection method starts by specifying an interval $[a, b]$ such that $f(a)f(b) < 0$, that is, an interval inside of which there is a sign change in $f(x)$. Given such an interval, we proceed as follows: Compute the middle point of the interval $m = (a + b)/2$ and shorten the interval to $[a, m]$ or to $[m, b]$ if, respectively, $f(a)f(m) < 0$ or $f(m)f(b) < 0$. Repeat the preceding procedure until the size of the interval is less than or equal to a desired precision. Below is a pseudocode:

```
Inputs:
 function f(x);
 a and b such that f(a)f(b) < 0;
 desired precision ϵ

while (b − a) > ϵ do
  m ← (a + b)/2
  if f(a)f(m) < 0 then
    b ← m
  else
    a ← m
  end if
end while

Result: a is an approximation to the root of f(x), f(a) ≈ 0.
```

Question 4:

Consider the directed and weighted graph given below as an adjacency list.

A → (B, 4), (C, 2), (F, 7)

B → (A, 4), (C, 5), (D, 10), (G, 3), (K, 5)

C → (A, 2), (B, 5), (D, 3), (E, 4), (F, 6), (K, 1)

D → (B, 10), (C, 3), (E, 1), (H, 8)

E → (C, 4), (D, 1), (H, 2), (I, 6)

F → (A, 7), (C, 6), (G, 2), (J, 5), (M, 4)

G → (B, 3), (F, 2), (H, 4), (J, 7), (M, 1)

H → (D, 8), (E, 2), (G, 4), (I, 3), (L, 1)

I → (E, 6), (H, 3), (J, 4), (L, 5)

J → (F, 5), (G, 7), (I, 4), (M, 6)

K → (B, 5), (C, 1)

L → (H, 1), (I, 5)

M → (F, 4), (G, 1), (J, 6)

Using Dijkstra's algorithm, find the minumum path from K to L. Give the minimum distance and the corresponding path (sequence of vertices). Show the steps of the algorithm.

Space to answer Question 4 (page 1):

Minimum distance: K → L: 8

Shortest path: K → C → E → H → L

Weights on the path: K–C (1), C–E (4), E–H (2), H–L (1) → 1 + 4 + 2 + 1 = 8

Steps of the algorithm:

Iteration	Action	u	d(u)	Finalized	updated
0	finalize K	K	0	K	C=1; B=5
1	finalize C	C	1	C, K	D=4; E=5; F=7
2	finalize D	D	4	C, D, K	–
3	finalize B	B	5	B, C, D, K	G=8
4	finalize E	E	5	B, C, D, E, K	H=7
5	finalize H	H	7	B, C, D, E, H, K	L=8
6	finalize L	L	8	B, C, D, E, H, K, L	–

Steps of the algorithm

Inicialization: d(K)=0; for the other vertices d=∞. Lists of predecessors empty.

Main iterations (heap extraction order and relavant relaxations):

Finaliz2 K (d=0)
Relax:
K→C (1): d(C)=1, prev(C)=K
K→B (5): d(B)=5, prev(B)=K

Finalize C (d=1)
Relax:
C→D (3): d(D)=4, prev(D)=C
C→E (4): d(E)=5, prev(E)=C
C→F (6): d(F)=7, prev(F)=C
(C→A, C→B, C→K don't improve)

Finalizw D (d=4)
Relax:
D→E (1): d(E)=min(5,4+1)=5 (stay at 5)
(D→C, D→B, D→H don't improve)

Finalize B (d=5)
Relaxa:
B→G (3): d(G)=8, prev(G)=B
(B→A, C, D, K não melhoram)

Finalize E (d=5)
Relax:
E→H (2): d(H)=7, prev(H)=E

Finalize H (d=7)
Relax:
H→L (1): d(L)=8, prev(L)=H ← destination reached
(H→D, E, G, I don't give useful updates to reach L)
Finaliza L (d=8) → stop (arrived at the destination)

Path reconstruction using the predecessors:
L ← H ← E ← C ← K → caminho K–C–E–H–L.

Alternative correct answer:

A possible execution (with tie resolution favoring D over E):

Finalize K (0): get d(C)=1, d(B)=5.
Finalize C (1): get d(D)=4, d(E)=5, d(F)=7.
Finalize D (4): relax D→E com custo 4+1=5 (tie), use prev(E)=D.
Finalize E (5): get d(H)=7 (prev(H)=E).
Finalize H (7): get d(L)=8 (prev(L)=H).
Finalize L (8): reconstruct K ← C ← D ← E ← H ← L → K–C–D–E–H–L.

Question 5:

Consider the directed and unweighted graph given below as an adjacency list.

A → B, C, F

B → A, C, D, G, K

C → A, B, D, E, F, K

D → B, C, E, H

E → C, D, H, I

F → A, C, G, J, M

G → B, F, H, J, M

H → D, E, G, I, L

I → E, H, J, L

J → F, G, I, M

K → B, C

L → H, I

M → F, G, J

Use Breadth First Search to find the distance between vertices A and M. Show the sequence of vertices in the minimum path and the steps in the algorithm.

Space to answer Question 5 (page 1):

Steps (BFS levels)

Level 0: A
Found: B, C, F (parent: A)

Level 1: B, C, F
From F we find M (via F–M).
As soon as M is found, we stop.

Reconstrução do caminho

Parents: F ← A, M ← F
⇒ Path: A → F → M