University of São Paulo

São Carlos Institute of Physics

Graduate Program

Admission Test

Computational Physics

Second Semester 2025

Exam Booklet

Candidate's Code:

Physics Questions
(Multiple Choice)

Instructions: The Physics questions are multiple-choice. For these questions, please indicate your chosen answer directly in this exam booklet by marking the corresponding square with an "X" using a black or blue pen. Do not use this exam booklet for elaborating on your answers or as a draft. Additionally, you will receive a separate notepaper for drafting and calculations. **The answers considered for correction will be the ones marked in the exam booklet.**

Question 1:

An atom with mass m = 4u (where u is the atomic mass unit) moves in one dimension x, in the positive x region, under the influence of an interaction potential. The potential energy of this system is given by:

$$U(x) = A\left(\frac{a^3}{x^3} - \frac{a}{x}\right)$$

The atom is released at rest from the position x = 1 nm.
[Use A = 20 u $(nm/s)^2$, a = 1 nm, where u is the atomic mass unit and nm = 10^{-9} m.]

Place an ‘X’ in the square that corresponds to the correct answer.

☐ The potential has a stable equilibrium point at x = 1 nm. The magnitude of the atom's velocity at x = 2a is $v(2a) = \frac{\sqrt{7}}{2} nm/s$.

☐ The potential has a stable equilibrium point at $x = \sqrt{3}\, nm$. The magnitude of the atom's velocity at x = 2a is $v(2a) = 3.5\, nm/s$.

☐ The potential has an unstable equilibrium point at $x = \sqrt{3}\, nm$. The magnitude of the atom's velocity at x = 2a is $v(2a) = \sqrt{3}\, nm/s$.

☒ The potential has a stable equilibrium point at $x = \sqrt{3}\, nm$. The magnitude of the atom's velocity at x = 2a is $v(2a) = \frac{\sqrt{15}}{2}\, nm/s$.

☐ The potential has an unstable equilibrium point at $x = 1\, nm$. The magnitude of the atom's velocity at x = 2a is $v(2a) = \frac{\sqrt{5}}{4}\, nm/s$.

Place an ‘X’ in the square that corresponds to the correct answer.

The potential is conservative because the problem is one-dimensional and U(x) depends only on position. Therefore, the total energy is conserved, and we have U(x = 1 nm) = 0 u $(nm/s)^2$. The condition for extrema, dU/dx = 0, gives a critical point at $x = \sqrt{3}\,nm$, and from the second derivative, which is positive, we conclude that it is a stable equilibrium point.

To find |v(2a)|, we use:

E_c = (1/2) mv^2 = U(1 nm) - U(2a) = (3/8) A,

which gives v^2 = (3/4) A /m = (3/4) (20/4) = (3/4)5 nm/s

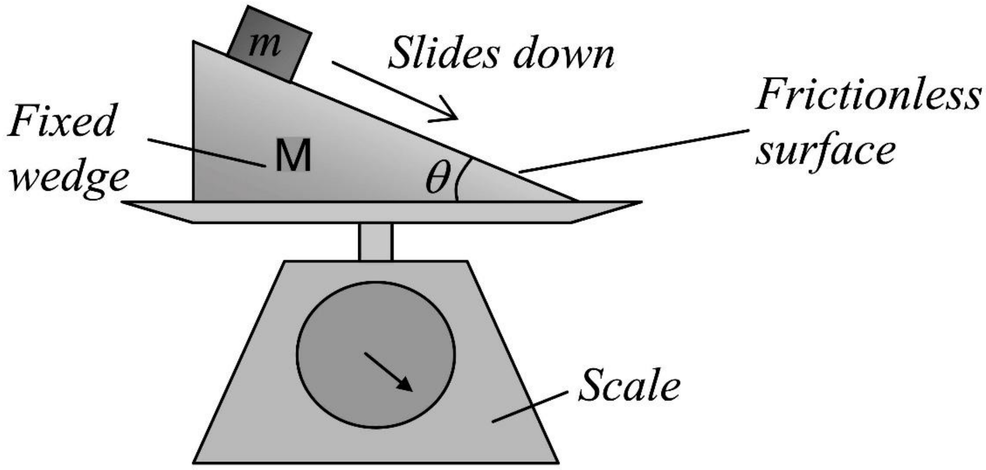
So,

$$v = \frac{\sqrt{15}}{2}\ nm/s$$

Question 2:

A block of mass m = 1kg slides without friction on a wedge of mass M = 2kg, which is fixed on a scale (as depicted in the figure). The angle in the figure is such that tan θ=3/4. [If necessary, use $g = 10 m/s^2$.]

Whilst the block slides on the wedge, what does the scale read?



Place an 'X' in the square that corresponds to the correct answer.

☐ (14/5) kg
☐ (13/5) kg
☐ (59/25) kg
☐ (41/25) kg
X (66/25) kg

The block, with mass m = 1 kg, exerts a normal force on the inclined surface of the wedge given by

$N = \mu g \cos(\theta)$.

We know that $\cos(\theta) = 4/5$ because the wedge has the shape of a 3-4-5 right triangle.

The wedge, therefore, experiences a vertical downward force given by

$N_v = m g \cos^2(\theta)$.

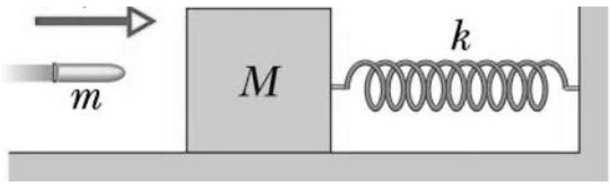
Thus, the total normal force on the scale will be
$g [M + m \cos^2(\theta)]$,

and the scale will read

$M + m \cos^2(\theta) = 2 \text{ kg} + 1 \text{ kg} * 16/25 = (66/25) \text{ kg}$.

Question 3:

A block of mass M=3.990 kg, initially at rest on a frictionless horizontal table, is attached to a rigid support by a spring with a spring constant k=3600 N/m. A bullet of mass m=10 g and velocity v of magnitude 600 m/s, as shown in the figure below, strikes and becomes embedded in the block. Assuming that the compression of the spring is zero until the bullet is completely embedded in the block, the speed of the block + bullet system v_s immediately after the collision, and the amplitude A of the resulting simple harmonic motion, are respectively:



Place an 'X' in the square that corresponds to the correct answer.

X v_s = 1.5 m/s e A = 5 cm
☐ v_s = 1.5 m/s e A = 10 cm
☐ v_s = 3 m/s e A = 5 cm
☐ v_s = 3 m/s e A = 10 cm
☐ None of the previous answers

The conservation of linear momentum in the collision of the bullet with the block implies that the velocity of the system: bullet + block immediately after the collision is:

$$mv = (M + m)v_S$$

That is,

$$v_S = \frac{mv}{(M + m)}.$$

After the collision, the mechanical energy is conserved. The amplitude of the harmonic motion is determined by the point at which the total mechanical energy is completely stored as elastic potential energy,

$$E = \frac{(M + m)}{2}\frac{m^2}{(M + m)^2}v^2 = \frac{1}{2}kA^2.$$

Solving for A,

$$A = \frac{mv}{\sqrt{k(M + M)}}.$$

Using the numerical values, we find $v_s = 1.5m/s$ e $A = 5cm$.

Question 4:

A mechanical motor causes a very long wire to vibrate up and down, producing propagating waves. At the far end of the wire, the traveling waves are absorbed, so there is no reflection process. The speed of the generated wave is 240 m/s, the maximum transverse displacement of the wire is 1 cm, and the distance between consecutive maxima is 3.0 m. Considering y(x=0,t=0)=0, the wave function (in SI units) that represents the wave propagating along this wire and the maximum transverse velocity v_m of a point on the wire are, respectively:

Place an 'X' in the square that corresponds to the correct answer.

☐ $y(x,t) = 0.02\, sen(2\pi x/3 - 160\pi\, t)$ e $v_m = 1.6\,\pi\, m/s$

☐ $y(x,t) = 0.01\, sen(2\pi x/3 - 80\pi\, t)$ e $v_m = 3.2\,\pi\, m/s$

X $y(x,t) = 0.01\, sen(2\pi x/3 - 160\pi\, t)$ e $v_m = 1.6\,\pi\, m/s$

☐ $y(x,t) = 0.01\, sen(2\pi x/3 - 160\pi\, t)$ e $v_m = 3.2\,\pi\, m/s$

☐ None of the previous answers

We want to write $y(x,t) = A\,sin(kx - \omega t + \varphi)$. According to the initial condition, $\varphi = 0$ and the amplitude is $A = 0.01\,m$. To determine k, we use $k = 2\pi/\lambda = (2\pi/3)m^{-1}$ while $v = \lambda f$, which leads to $f = 80Hz$. From this, $\omega = 160\pi\,rad/s$. Combining these results, we arrive at

$$y(x,t) = 0{,}01\,sin(2\pi x/3 - 160\pi\,t).$$

The maximum transverse velocity v_m is determined from the maximum of the derivative of $y(x,t)$ with respect to time, which corresponds to $v_m = A\omega = 1.6\pi\,m/s$.

Question 5:

A copper piece with 300 g is heated at temperature T in an oven. The copper is then introduced in a 150 g copper calorimeter with 220 g of water. The initial temperature of the water and calorimeter was 20 ^{o}C and the temperature after the equilibrium is 100 ^{o}C. The final weight of the calorimeter and its contents indicate that 5 g of water was lost due to evaporation. What is the temperature T? Consider: Specific heat capacity: water=1.0 cal/g.K, copper=0.09 cal/g.K. Latent heat of vaporization: water=540 cal/g.

Place an 'X' in the square that corresponds to the correct answer.

☒ 892 ^{o}C.
☐ 877 ^{o}C
☐ 692 ^{o}C
☐ 677 ^{o}C
☐ None of the proposed answers.

$Q_{calorimeter} = m_{alum}\ c_{cupper}(T_f - T_i) + m_{water}\ c_{water}(T_f - T_i) + m_{water\ vapor}\ Q_{water\ vapor}$

$Q_{calorimeter} = 150 * 0.09 * (100 - 20) + 220 * 1 * (100 - 20) + 5 * 540$

$Q_{calorimeter} = 1080 + 17600 + 2700 = 21380$ cal

$Q_{Hot\ cupper} = m_{Hot\ cupper} * c_{cupper} * (T - T_f)$

$Q_{Hot\ cupper} = 300 * 0.09 * (T - 100) = 27 * (T - 100) = 27\ T - 2700$

Equating the two equations:

$27\ T - 2700 = 21380$

$27\ T = 21380 + 2700 = 24080$

$T = 892\ ^{o}C$

Computing Science Questions
(Open Answers)

Instructions: All questions related to computer science are in the form of open-ended questions. Therefore, you are expected to provide a comprehensive response for each question in the designated space within this examination booklet. Please use a black or blue pen for your responses. Additionally, you will receive a separate notepaper for drafting and calculations, but **only the answers provided within the allocated space in the exam booklet will be considered for evaluation.**

Question 1:

Write a function named translate with three strings as parameters, called text, schar and dchar. The strings schar and dchar are lists of characters. The function should change the characters in text in the following way: if a character is in schar, it should be substituted by the character in the corresponding entry in dchar. If the character is not in schar it should not be altered. If schar and dchar are not of the same size, all additional characters in the largest of the two should be ignored.

As an example, a call to the function with "abobora" for text, "aeo" for schar and "oiua" for dchar results in "obuburo" (all the a were turned to o, all the o to u; the b and r remained unchanged; the a in dchar was ignored).

Use C, C++, Fortran or Java in your code.

Space to answer Question 1 (page 1):

```
char *translate(const char *text, const char *schar, const char *dchar) {
  size_t nt = strlen(text);
  char *newtext = (char *)malloc(nt + 1);
  size_t ns = strlen(schar);
  size_t nd = strlen(dchar);
  size_t n = ns < nd ? ns : nd;
  for (size_t i = 0; i < nt; ++i) {
    newtext[i] = text[i];
    for (size_t j = 0; j < n; ++j) {
      if (text[i] == schar[j]) {
        newtext[i] = dchar[j];
      }
    }
  }
  newtext[nt] = '\0';
  return newtext;
}
```

Question 2:

Triangular numbers are generated by

$$T(n) = n\,(n+1)/2,$$

with $n \geq 1$ *integer*. Similarly pentagonal numbers $P(n)$ and hexagonal numbers $H(n)$ are generated by

$$P(n) = n\,(3n-1)/2\,,$$
$$H(n) = n(2n-1),$$

with $n \geq 1$ *integer*.

Note that $T(1) = P(1) = H(1) = 1$, e therefore 1 is simultaneously triangular, pentagonal, and hexagonal.

Write a program to find a number greater than 1 that is simultaneously triangular, pentagonal, and hexagonal, i.e., find N such that:

$N = T(a) = P(b) = H(c)$, for some integers de a, b e c larger than 1.

The program should print the values of N, a, b and c.

Use C, C++, Fortran or Java in your code.

Space to answer Question 2 (page 1):

```
#include <stdio.h>
#include <stdbool.h>

int main()
{
  int a = 1, b = 2, c = 1;
  int ta, pb, hc;

  while (true) {
    ta = a * (a + 1) / 2;
    pb = b * (3 * b - 1) / 2;
    hc = c * (2 * c - 1);
    if (ta == pb && ta == hc) break;
```

```
    if (ta <= pb && ta <= hc) ++a;
    else if (pb <= ta && pb <= hc) ++b;
    else if (hc <= ta && hc <= pb) ++c;
  }
  printf("T(%d) = P(%d) = H(%d) = %d\n", a, b, c, ta);

  return 0;
}
```

Question 3:

Given a set of three pair of experimental values of a function:

$y_1 = f(x_1), y_2 = f(x_2), y_3 = f(x_3)$.

a) For a polynomial interpolation, which would the largest meaningful order that could be used?
b) Describe a process/algorithm to construct the polynomial from the data.
c) Employ this procedure for the following set of values:
   $x_1 = 1, y_1 = 10, x_2 = 3, y_2 = 36, x_3 = 4, y_3 = 55$.

Space to answer Question 3 (page 1):

a) The largest meaningful order is 2, as we are given 3 data points, which are enough to fix the three coefficients of orders 0 to 2.

b) One way to construct a polynomial from the data is to use Lagrange's polynomial: $P(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3$
   Other methods, as e.g. Neville's algorithm, are possible.

c) The resulting polynomial is $2x^2 + 5x + 3$.

Question 4:

Consider an initially empty AVL tree. The following values are to be inserted in sequence:

30, 20, 40, 10, 25, 35, 50, 5, 27, 28, 3

Do the following operations:

a) Insert the values in the given order, taking care that the tree remains balanced after each insertion. For each insertion that causes an inbalance, describe the (single or double) rotation that was applied to restore the balance. After each insertion draw the resulting AVL tree with the balance factor for each node.

b) Remove the node with value 20 from the resulting tree. Explain, step by step, how the removal afects the balance and which rotations are necessary to restore the properties of the AVL tree. Draw the tree after the removal with the new balance factor of each of the remaining nodes.

Space to answer Question 4 (page 1):

a)

```
              30

          /        \
        20         40
      /    \     /  \
     5     27    35  50
    / \   /  \
   3  10 25  28
```

b)

```
              30
         /         \

        25          40

      /   \        /  \

     5    27       35   50

    / \      \

   3  10      28
```

Question 5:

Consider the following weighted undirected graph:

Vertices:

V = {A, B, C, D, E, F, G, H}

Edges, with weights:

E = {
(A, B, 2), (A, C, 4), (B, C, 1), (B, D, 7),
(C, E, 3), (D, F, 1), (E, D, 2), (E, F, 5),
(E, G, 1), (F, H, 3),(G, H, 2)
}

Using Dijkstra's algorithm, find the minimum path from vertex A to vertex H. In your answer, present the steps of the algorithm, showing the visited vertices and the updates to the minimum costs. Give the minimum path and its cost.

Space to answer Question 5 (page 1):

Minimum cost path:

A → B → C → E → G → H

Total cost:

2 (A→B) + 1 (B→C) + 3 (C→E) + 1 (E→G) + 2 (G→H) = 9