University of São Paulo

São Carlos Institute of Physics

Graduate Program

Admission Test

Computational Physics

First Semester 2025

Exam Booklet

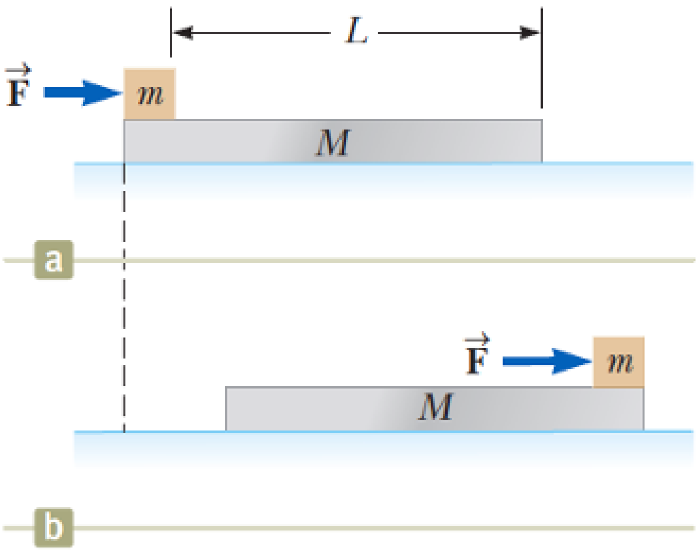
Candidate's Code:

Physics Questions
(Multiple Choice)

Instructions: The Physics questions are multiple-choice. For these questions, please indicate your chosen answer directly in this exam booklet by marking the corresponding square with an "X" using a black or blue pen. Do not use this exam booklet for elaborating on your answers or as a draft. You may use the provided Notepaper for developing your answers or as a draft paper. **The answers considered for correction will be the ones marked in the exam booklet.**

Question 1:

A block of mass *m* is at rest at the left end of a block of mass *M*, which is also at rest, as shown in the figure below. The distance between the edge of the two blocks is L, as indicated in the figure. The coefficient of kinetic friction between the two blocks is $\mu = 1/2$, and the surface on which the block of mass *M* rests is frictionless. A constant horizontal force is applied to the block of mass *m*, setting it in motion as shown in the figure. Knowing that the force $F = 2mg$ and that $m = M/3$ (where g is the local acceleration gravity), when the block of mass *m* reaches the end of the block of mass *M* (situation b in the figure), the edge of the block of mass *M* will have moved from its initial position by a distance D given by:



Place an 'X' in the square that corresponds to the correct answer.

☐ D = L/22

☒ D = L/8

☐ D = L/26

☐ D = L/4

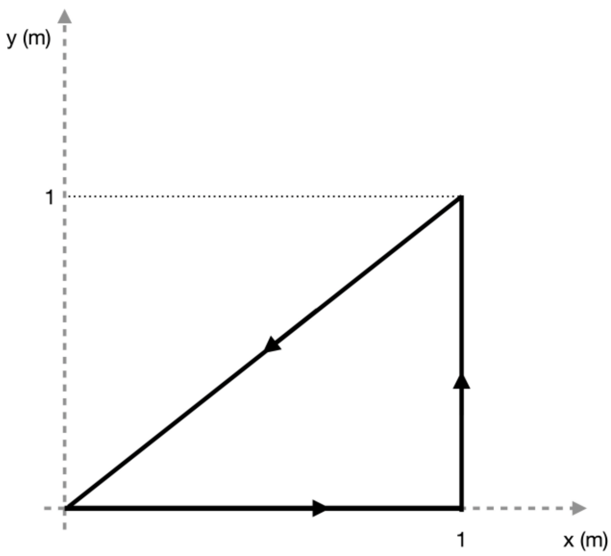
☐ D = L/3

The block of mass *m* has an acceleration given by $a_m = (F - Fat)/m = (F - \mu m g)/m$. The acceleration of the block of mass *M* is $a_M = (m/M)\mu g$. Both have the equation of motion $x(t) = (1/2) a t^2$, with their respective accelerations. Thus, the block of mass *m* takes a time $(t_f)^2 = 2L/(am - aM)$ to reach the end of the block of mass *M*. During this time, the block of mass M will travel a distance given by, using the data from the problem,

$D = x(t_f) = (1/2) (m/M)\mu g \, 2L/(am - aM) = L/8$

Question 2:

A particle moves along the closed path in the xy-plane represented in the figure, starting its motion at the origin and performing one complete turn along this path, under the action of a force $\mathbf{F_1} = C\,(y^2\,\boldsymbol{i} - 2xy\,\boldsymbol{j})$, with C = 10 J/m². Another particle follows the same path under the action of a different force $\mathbf{F_2} = C\,(y^2\,\boldsymbol{i} + 2xy\,\boldsymbol{j})$. We are interested in the work W1 and W2 done by the forces $\mathbf{F_1}$ and $\mathbf{F_2}$, respectively, along the closed path shown in the figure, and in what can be concluded based only on the results obtained for W1 and W2.



Place an ‘X’ in the square that corresponds to the correct answer.

☐ W1 = 0J, W2=0J. The two forces may be conservative

☐ W1 = 0J, W2 = -10/3J. The force $\mathbf{F_1}$ is conservative.

☒ W1= -20/3 J, W2=0J. The force $\mathbf{F_2}$ may be conservative.

☐ W1= -20/3 J, W2=0J. The force $\mathbf{F_2}$ is conservative.

☐ W1= -20/3 J, W2=+10J. None of the forces is conservative.

We will split the calculation into 3 segments. The first segment goes from the origin to the point $(x, y) = (1, 0)$, the second from $(1, 0)$ to $(1, 1)$, and the third from $(1, 1)$ back to the origin, always following the path indicated in the figure. We denote the work done by force i in segment j by $W_{F_i}^{(j)}$. Recall that $\vec{F}_1 = C(y^2\hat{\imath} - 2xy\hat{\jmath})$ and $\vec{F}_2 = C(y^2\hat{\imath} + 2xy\hat{\jmath})$, with $C = 10\,\mathrm{J/m^2}$.

Thus, in segment 1, we see that both forces are zero (since $y = 0$) and

$$W_{F_1}^{(1)} = W_{F_2}^{(1)} = 0\,\mathrm{J}.$$

In segment 2, we have $x = 1$ and $d\vec{\ell}_2 = dy\hat{\jmath}$, and for the work done,

$$W_{F_1}^{(2)} = \int \vec{F}_1 \cdot d\vec{\ell}_2 = C\int_0^1 (-2y\,dy) = -10\,\mathrm{J},$$

and for force 2,

$$W_{F_2}^{(2)} = \int \vec{F}_2 \cdot d\vec{\ell}_2 = C\int_0^1 (+2y\,dy) = +10\,\mathrm{J}.$$

In segment 3, we have $y = x$ and $dx = dy$. Thus,

$$\begin{aligned} W_{F_1}^{(3)} &= C\int_1^0 x^2\,dx - C\int_1^0 2x^2\,dx = +\frac{10}{3}\,\mathrm{J}, \\ W_{F_2}^{(3)} &= C\int_1^0 x^2\,dx + C\int_1^0 2x^2\,dx = -10\,\mathrm{J}. \end{aligned}$$

For the total work, we have

$$\begin{aligned} W_{F_1} &= \sum_{j=1}^{3} W_{F_1}^{(j)} = -10\,\mathrm{J} + \frac{10}{3}\,\mathrm{J} = -\frac{20}{3}\,\mathrm{J}, \\ W_{F_2} &= \sum_{j=1}^{3} W_{F_2}^{(j)} = 0\,\mathrm{J}. \end{aligned}$$

It is possible to conclude that the force $\vec{F}_1$ is not conservative and that the force $\vec{F}_2$ may be conservative (we cannot be certain based only on the calculation of the work for a specific closed path).

Question 3:

An object oscillates with angular frequency ω= 5.0 rad/s. At t = 0 s, the object is at x = 10.0 cm with an initial velocity v_x = - 50 cm/s. Find the phase constant of the motion e x as a function of time.

Place an 'X' in the square that corresponds to the correct answer.

☐ $\pi/8$ rad; x = (5.0 cm) cos[(10.0 s^{-1})t + $\pi/2$]

☐ $\pi/8$ rad; x = (20.0 cm) [sen(5.0 s^{-1} t + $\pi/4$)]

☒ $\pi/4$ rad; x = (10.0 cm) [cos(5.0 s^{-1} t) - sen(5.0 s^{-1} t)]

☐ $\pi/4$ rad; x = (4.0 cm) [cos(5.0 s^{-1} t) + sen(5.0 s^{-1} t)]

☐ None of the previous answers

The initial position and velocity are related to the amplitude and phase constant.

$x = A\cos(\omega t + \delta)$ e $v_x = -\omega A\,\mathrm{sen}(\omega t + \delta)$

Then,

$x_0 = A\cos(\delta)$

and

$v_{0x} = -\omega A\,\mathrm{sen}(\delta)$.

Thus,

$v_{0x}/x_0 = -\omega A\,\mathrm{sen}(\delta) / A\cos(\delta) = -\omega\tan(\delta)$.

Therefore,

$\tan(\delta) = -v_{0x}/\omega x_0$

$\delta = \tan^{-1}(-v_{0x}/\omega x_0) = \tan^{-1}(-(-50\text{ cm/s})/(5.0\text{ rad/s})(10.0\text{ cm})) = \pi/4$ rad

$A = x_0 / \cos(\delta) = 10.0\text{ cm}/\cos(\pi/4) = 20.0/\sqrt{2}$ cm

$x = (20{,}0/\sqrt{2}\text{ cm})\cos[(5.0\text{ s}^{-1})t + \pi/4] = 10.0[\cos(5.0\text{ s}^{-1} t) - \mathrm{sen}(5.0\text{ s}^{-1} t)]$

Question 4:

Consider a harmonic wave function in a string:

Y(x,t) = (0.25 m) sen[(2.0 m-1)x – (8.0 s^{-1})t]

Find the wavelength, the frequency and the maximum speed at any point in the string.

Place an 'X' in the square that corresponds to the correct answer.

☐ 3π /4 m; 1/π s; 1.0 m/s

X π m; 4/π s; 2.0 m/s

☐ 2.0 m; 1.5 s; 3.0 m/s

☐ 4.0 m; 2.0 s; 0.5π m/s

☐ None of the previous answers

$\lambda = 2\pi/k = 2\pi/2.0\ m^{-1} = \pi\ m$

$T = 2\pi/\omega = 2\pi/8.0\ s^{-1} = \pi/4\ s$

$f = 1/T = 4/\pi\ s$

$A = 0.25\ m = ¼\ m$

$v_y = (0.25\ m)\ (-8.0\ s^{-1})\ \cos[(2.0\ m^{-1})x - (8.0\ s^{-1})t] = -(2.0\ m/s)\ \cos[(2.0\ m^{-1})x – (8.0\ s^{-1})t]$

$v_{y,max} = 2.0\ m / s$

Question 5:

Suppose that you have no heat source to heat a jar of water (500 ml) at initial temperature of 15 ^{o}C to make a coffee. You can heat water by shaking it inside a thermos flask. Considering that in each shake the water falls 30 cm and all mechanical energy is converted into heat. If you are able to making 30 shakes each minute, neglecting any loss of thermal energy by the flask, how long you must shake the flask before the water boils.

Place an 'X' in the square that corresponds to the correct answer.

☒ 2.7 days

☐ 4.6 hours

☐ 39.5 hours

☐ 3.2 days

☐ None of the proposed answers.

Each shake of the bottle provides: $E = mgh$ = 0.5 kg*10 m/s^2*0.3 m=1.5 J

Every minute there are 30 shakes: E=30*1.5 J = 45 J/min

To almost boil 500 ml of water: $Q = mc(T_{final} - T_{inicial}) = mc\Delta T$

Q(cal)=500g*1 cal/g*(100-15)=500*1*85 = ~42500 cal

E(J)=42500 cal*4.18 J=~177650 J

Therefore, the time it takes to shake the jar is: 177650/45 = ~3948 min

3948 min = ~65.8 h = ~2.7 dias

Computing Science Questions
(Open Answers)

Instructions: All questions related to computer science are in the form of open-ended questions. Therefore, you are expected to provide a comprehensive response for each question in the designated space within this examination booklet. Please use a black or blue pen for your responses. Additionally, you will receive a separate notepaper for drafting and calculations, but **only the answers provided within the allocated space in the exam booklet will be considered for evaluation.**

Question 1:

The Collatz sequence of a positive integer *n* is given by the iteration

$c_0 = n,$

$c_i = \frac{c_{i-1}}{2}$, for $i \geq 1$ if c_{i-1} is even,

$c_i = 3c_{i-1} + 1$, for $i \geq 1$ if c_{i-1} is odd.

which is repeated until a value $c_i = 1$, is reached, when the sequence ends.

For example, the Collatz sequence of 13 is

13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Write a program that asks the user for a positive integer, verifies the validity of the number given, and then prints the Collatz sequence of this number, including the original value and the final 1

Use C, C++, Fortran or Java in your code.

Space to answer Question 1 (page 1):

After reading and verifying the initial value given by the user, it suffices to implement a loop computing the iteration given above, taking care of being sure that both the initial value and the ending 1 are printed, as requested.

An example solution in C:

```
#include <stdio.h>
int main(void) {
  int n;
  printf("Give the initial value: ");
  int read = scanf(" %d", &n);
  if (read != 1) {
    fprintf(stderr, "Invalid entry for an integer.\n");
    return 1;
  }
  if (n < 1) {
    fprintf(stderr, "%d is not a positive number\n", n);
```

```
    return 2;
  }
  while (n != 1) {
    printf("%d\n", n);
    if (n % 2 == 0)
      n /= 2;
    else
      n = 3 * n + 1;
  }
  printf("%d\n", n);
  return 0;
}
```

Question 2:

The proper divisors of a positive integer are all positive integers smaller than it that evenly divide it. For example, the proper divisors of 12 are 1, 2, 3, 4, and 6; the proper divisors of 15 are 1, 3 and 5.

Write a function that receives two positive integers *a* and *b* and find the product of all proper divisors common to a and b. For example, if *a* = 12 and *b* = 15, the function should return 3 (the product of 1 and 3).

Use C, C++, Fortran or Java in your code.

Space to answer Question 2 (page 1):

Just scan positive integers less than the smaller between a and b and multiply all that are divisors of both a and b.

See the example code in C below:

```
int common_proper_divisors_product(int a, int b) {
  int m = a < b ? a : b;
  int prod = 1;
  for (int i = 2; i < m; ++i) {
    if (a % i == 0 && b % i == 0) prod *= i;
  }
  return prod;
}
```

Space to answer Question 2 (page 2):

Question 3:

Write the expression of the complex exponential function in terms of time *t* for frequency *f* = 10Hz, zero phase, and unit amplitude. Present also the expressions of its real and imaginary parts. What is the relationship between this function and the continuous Fourier transform?

Space to answer Question 3 (page 1):

$g(t) = exp(i20\pi t)$
$\Re g(t) = cos(20\pi t)$
$\Im g(t) = sin(20\pi t)$

This function is employed for the definition of the basis of the continuous Fourier transform.

Space to answer Question 3 (page 2):

Question 4:

Given the graph defined by the set of vertices V and weighted edges E below:

V = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

E = { (0, 5, 343), (0, 8, 464), (0, 7, 1435), (1, 5, 879), (1, 6, 954), (1, 7, 811), (1, 9, 524), (2, 4, 1364), (2, 5, 1054), (3, 6, 433), (3, 9, 1053), (4, 5, 1106), (4, 9, 766), (6, 7, 837) }

Present the corresponding minimum spanning tree.

Space to answer Question 4 (page 1):

V = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

E = { (0, 5, 343), (3, 6, 433), (0, 8, 464), (1, 9, 524), (4, 9, 766), (1, 7, 811), (6, 7, 837), (1, 5, 879), (2, 5, 1054) }

Space to answer Question 4 (page 2):

Question 5:

Let K be a binary search tree whose elements were inserted in the following order: 45, 30, 60, 20, 35, 50, 70, 15, 25, 40, 55, 65, 75.

What are the order of traversal of the nodes in K using in-order, pre-order and post-order traversals?

Space to answer Question 5 (page 1):

In-order traversal: 15 20 25 30 35 40 45 50 55 60 65 70 75

Pre-order traversal: 45 30 20 15 25 35 40 60 50 55 70 65 75

Post-order traversal: 15 25 20 40 35 30 55 50 65 75 70 60 45

Space to answer Question 5 (page 2):