University of São Paulo

São Carlos Institute of Physics

Graduate Program

Admission Test

Computational Physics

Second Semester 2024

Answer Key

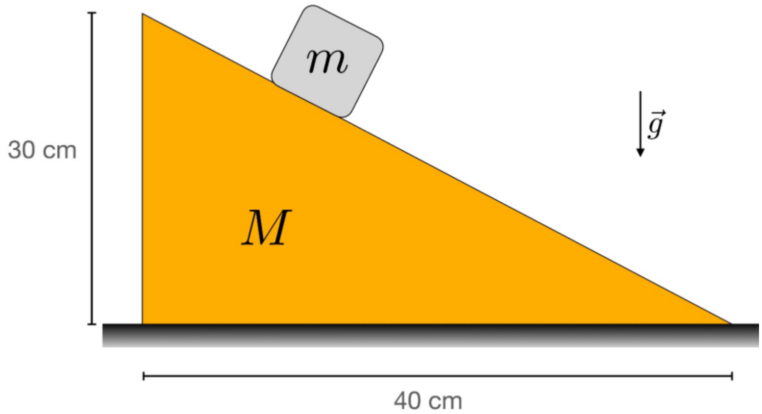
Candidate's Code:

Physics Questions
(Multiple Choice)

Instructions: The Physics questions are multiple-choice. For these questions, please indicate your chosen answer directly in this exam booklet by marking the corresponding square with an "X" using a black or blue pen. Do not use this exam booklet for elaborating on your answers or as a draft. You may use the provided Notepaper for developing your answers or as a draft paper. **The answers considered for correction will be the ones marked in the exam booklet.**

Question 1:

A block of mass $m = 1kg$ slides without friction on a wedge of mass M, whose dimensions are represented in the figure. There is friction between the wedge and the surface on which it rests, and the coefficient of static friction is $\mu = 1/10$. What is the smallest value of M (in kg) such that the block slides without the wedge slipping to the left in the figure? If necessary, *use* $g = 10m/s^2$.



Place an 'X' in the square that corresponds to the correct answer.

☐ 114/35

☐ 4/3

☒ 104/25

☐ 24/5

☐ None of the proposed answers.

Answer:

The block of mass m = *1kg*, while sliding, exerts a normal force on the inclined surface of the wedge, which is given by (see figure below):

$N = \mu\ g \cos(\theta)$.

We know $\cos(\theta) = 4/5$, as the wedge has the shape of a 3-4-5 right triangle. Thus, the wedge experiences a horizontal force to the left, represented in the figure, of magnitude

$N_h = m\ g \cos(\theta) \sin(\theta)$

and another vertical force downward given by

$N_v = m\ g \cos^2(\theta)$.

The maximum static friction force is then given by

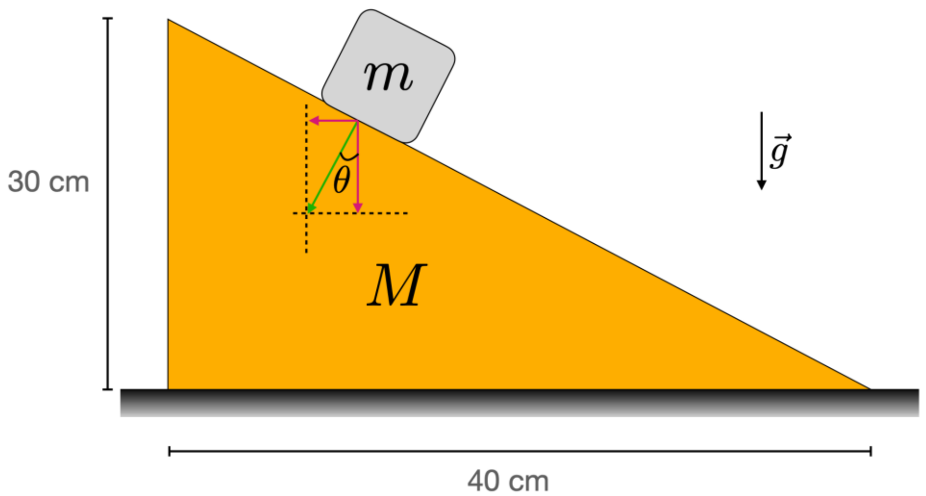
$F_{max} = \mu\ g\ [M + m \cos^2(\theta)]$,

since the normal reaction to the supporting plane is given by the weight of the wedge, Mg, added to the vertical component of the force exerted by the block on the wedge. In the situation where the mass M is at the minimum such that the wedge does not slide, we are in the situation of maximum static friction, therefore

$N_h = F_{max}$

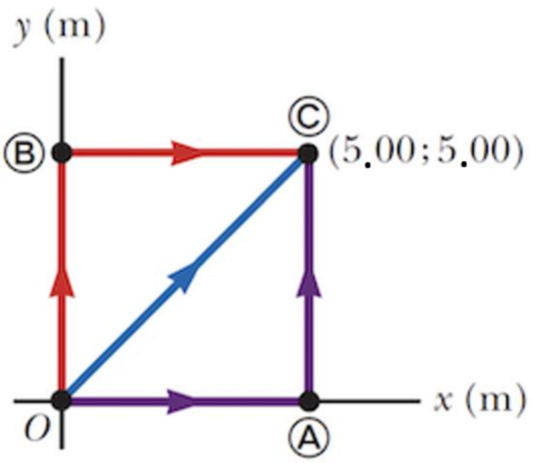
from which we obtain

$M_{min} = m \cos(\theta)\ (\sin(\theta)/\mu - \cos(\theta)) = 104/25$ kg.



Question 2:

A particle moves on the xy plane under the action of a force **F** = 2y**i** + x^2**j**, with F given in Newtons and x and y in meters. The particle moves from the origin to its final position at x=5.00m and y=5.00m (point C in the figure) following three different paths: OAC, OC, and OBC. We denote the work done by the force on the particle along each of these paths by W(OAC), W(OC) e W(OBC). About the work done in each of the paths it is correct that:



Place an 'X' in the square that corresponds to the correct answer.

☒ W(OBC)<W(OC)<W(OAC)

☐ W(OBC)<W(OAC)<W(OC)

☐ W(OBC)=W(OAC)=W(OC)

☐ W(OC)<W(OAC)<W(OBC)

☐ None of the proposed answers.

Answer:

1. W(OAC) = 125 J since in OA the work is zero (force is perpendicular to the trajectory) while in OC one has **F.dl** = x^2dy, with x fixed at 5.00m, and, therefore

W(OAC) = $\int_0^5 x^2 dy = 125J$.

2. W(OC) = 66.7 J. In this case the trajectory is such that y=x, and dy=dx. Then, **F.dl** = 2x dx + x^2dx. One finds for the work done by the force:

W(OC) = $\int_0^5 2x dx + \int_0^5 x^2 dx = 66.7J$.

3. W(OBC) = 50 J since the work on the path OB is zero (force is perpendicular to the trajectory) while in BC one has **F.dl** =2ydx with y=5.00m and, therefore:

W(OBC) = $\int_0^5 2y dx = 50J$.

Hence, the correct answer is W(OBC)<W(OC)<W(OAC).

Question 3:

An object oscillates with angular frequency ω= 8.0 rad/s. At t = 0 s, the object is at x = 4.0 cm with an initial velocity v_x = -25 cm/s. Find the amplitude, the phase constant, and x as a function of time.

Place an 'X' in the square that corresponds to the correct answer.

☒ 5.1 cm; 0.66 rad; x = (5.1 cm) cos[(8.0 s^{-1}) t + 0.66]

☐ 2.6 cm; 6.6 rad; x = (2.6 cm) cos[(4.0 s^{-1}) t + 6.6]

☐ 5.1 cm; 0.51 rad; x = (5.1 cm) cos[(4.0 s^{-1}) t + 0.51]

☐ 1.8 cm; 0.06 rad; x = (18.0 cm) cos[(8.0 s^{-1}) t + 0.86]

☐ None of the previous answers

Answer:

The initial position and velocity are related to the amplitude and phase constant.

$x = A\cos(\omega t + \delta)$ e $v_x = -\omega A\ \mathrm{sen}(\omega t + \delta)$

Thus,

$x_0 = A\cos(\delta)$

and

$v_{0x} = -\omega A\ \mathrm{sen}(\delta)$.

Therefore,

$v_{0x}/x_0 = -\omega A\mathrm{sen}(\delta)/A\cos(\delta) = -\ \omega\tan(\delta)$.

Consequently,

$\tan(\delta) = -v_{0x}/\omega_{x0}$

$\delta = \tan^{-1}(-v_{0x}/\omega_{x0}) = \tan^{-1}(-(-25\ \mathrm{cm/s})/(8.0\ \mathrm{rad/s})\ (4.0\ \mathrm{cm}))$
$= 0.66\ \mathrm{rad}$

$A = x_0/A\cos(\delta) = 4.0\ \mathrm{cm}/\cos(0.66) = 5.1\ \mathrm{cm}$

$x = (5.1\ \mathrm{cm})\cos[(8.0\ \mathrm{s}^{-1})t + 0.66]$

Question 4:

Consider the harmonic wave function on a string:

$Y(x,t) = (0.030\ m)\ sen[(2.2\ m^{-1})\ x - (3.5\ s^{-1})\ t]$

Find the wavelength, period, and maximum speed of any point on the string.

Place an 'X' in the square that corresponds to the correct answer.

☐ 0.4 m; 1.7 s; 1.1 m/s

☐ 5.8 m; 0.9 s; 3.5 m/s

X 2.9 m; 1.8 s; 0.11 m/s

☐ 0.3 m; 2.2 s; 0.35 m/s

☐ None of the previous answers

Answer:

The wave travels in the +x direction.

$v = \lambda/T = \omega/k = 3.5\ s^{-1}/2.2\ m^{-1} = 1.6\ m/s$

$\lambda = 2\pi/k = 2\pi/2.2\ m^{-1} = 2.9\ m$

$T = 2\pi/\omega = 2\pi/3.5\ s^{-1} = 1.8\ s$

$f = 1/T = 1/1.8\ s = 0.56\ Hz$

$A = 0.030\ m$

$v_y = (0.030\ m)\ (-\ 3.5\ s^{-1})\ \cos[(2.2\ m^{-1})\ x - (3.5\ s^{-1})\ t]$

$\quad = -\ (0.105\ m/s)\ \cos[(2.2\ m^{-1})\ x - (3.5\ s^{-1})\ t]$

$v_{y,max} = 0.11\ m/s$

Question 5:

A reservoir with a movable piston is in contact with a heated plate. Initially, this reservoir contained 1.00 kg of liquid water at 100°C, which was completely converted into vapor at 100°C by boiling at atmospheric pressure (1.01×10^5 Pa). The volume changed from 1.00×10^{-3} m^3 in the liquid state to 1.671 m^3 of water vapor only. Knowing that the heat of fusion for water is 333 kJ/kg and that of vaporization is 2260 kJ/kg, determine:

Determine W, the work done by the system during this process, Q, the amount of heat added in the process, and ΔU, the change of internal energy.

Place an 'X' in the square that corresponds to the correct answer.

☐ None of the proposed answers.

X W= 169 kJ; Q= 2260 kJ; ΔU= 2.09 MJ

☐ W= 68 kJ; Q= 2260 kJ; ΔU= 2.43 MJ

☐ W= 68 kJ; Q= 0 kJ; ΔU= 2.09 MJ

☐ W = 169 kJ; Q = 333 kJ; ΔU= 2.09 MJ

Answer:

1. As the pressure is constant during the boiling process, the work is calculated by the following equation, where the pressure can be placed outside the integral.

$$\mathrm{W} = -\int_{V_i}^{V_f} P\, dV = P\left(V_f - V_i\right) = 1{,}01\ \cdot 10^5\ Pa\ \cdot (1{,}671\, m^3 - 1{,}00 \cdot 10^{-3}\ m^3) = 169\ kJ$$

2. As there is no change in temperature, but only a change in phase, we use the heat of transformation related to the vaporization process:

$$Q = L \cdot m = 2260 \frac{kJ}{1{,}00} kg = 2260\ kJ$$

3. The change in internal energy is obtained by the first Law of Thermodynamics:

$$Q = \ \Delta U + W \rightarrow\ \Delta U = Q - W$$
$$\Delta U = 2260\ kJ - 169\ kJ = 2{,}09\ MJ$$

This quantity is positive, as the internal energy of the system has increased. It is less than the value added in the system because approximately 169 kJ of added heat was transformed into external work against atmospheric pressure.

Computing Science Questions
(Open Answers)

Instructions: All questions related to computer science are in the form of open-ended questions. Therefore, you are expected to provide a comprehensive response for each question in the designated space within this examination booklet. Please use a black or blue pen for your responses. Additionally, you will receive a separate notepaper for drafting and calculations, **but only the answers provided within the allocated space in the exam booklet will be considered for evaluation.**

Question 1:

Calculate the Fourier transform of the function

$$g(t) = 3\delta(t - t_o)$$

where $\delta(t)$ is the Dirac delta function. Specify the definition of the Fourier transform used, and maintain the intermediate steps of the development of the result.

Solution: Working with the following definition of the Fourier transform:

$$F(\omega) = \int_{-\infty}^{\infty} f\,(t) e^{i\omega t} dt$$

where ω is the angular frequency, and substituting $g(t)$ we get

$$G(\omega) = \int_{-\infty}^{\infty} g\,(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} 3\,\delta(t - t_0) e^{-i\omega t} dt = 3e^{-i\omega t_0}$$

due to the sampling property of the Dirac delta function.

Observations: Depending on the definition of the Fourier transform used, there may be a sign change of the exponent or the inclusion of a factor of $1/2\pi$ or $1/\sqrt{2\pi}$. Also, the transform can be expressed in terms of the frequency $f = \omega/2\pi$.

Question 2:

Write a function that takes an array (vector) of double-precision floating-point values and two parameters a and b, with $a \leq b$, and returns the average of all values in the array that fall within the interval $[a, b]$.

Use C, C++, Fortran or Java in your code.

Solution:
A possible solution in C is:

```
double sumPartial(double *v, size_t n, double a, double b)
{
    double s = 0;
    for (size_t i = 0; i < n; ++i) {
        if (v[i] >= a && v[i] <= b) {
            s += v[i];
        }
    }
    return s;
}
```

Question 3:

Considering a positive integer n, its divisors are the positive integers k (including n itself) that divide n without remainder. For example, the divisors of 12 are 1, 2, 3, 4, 6, and 12. The *sum of positive divisors* function $\sigma(n,x)$, for n a positive integer and x any real number, is defined as:

Use C, C++, Fortran or Java in your code.

Solution:
A possible solution in C is:

```
double sigma(int n, double x)
{
    double s = 1.0;
    for (int i = 2; i <= n; ++i) {
        if (n % i == 0) {
            s += pow(i, x);
        }
    }
    return s;
}
```

Question 4:

Consider an undirected graph G with the following connections between vertices:

(1,2), (1,6), (2,3), (2,4), (2,5), (3,4), (4,7), (5,6), (5,7), (6,7)

Part A:
Consider the following statements regarding the graph G:

I) G is bipartite.

II) G is a complete graph.

III) G has an Eulerian cycle.

IV) G is a planar graph.

For each statement, indicate whether it is true or false and explain.

Part B:
Consider that an arbitrary vertex v in G is removed, along with all its incident edges. As a result, the statements below are true or false? Explain.

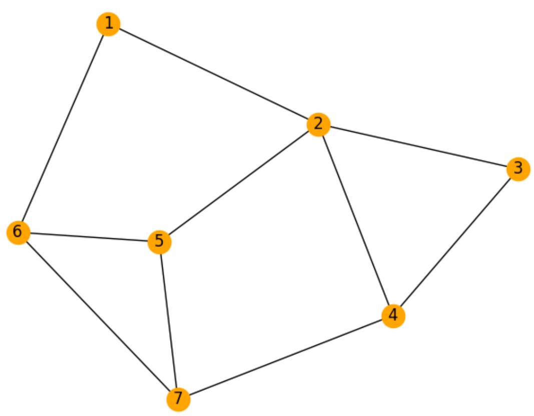
V) The resulting graph will be disconnected.

VI) The resulting graph will be planar.

VII) The resulting graph will have an Eulerian cycle.

VIII) The resulting graph will be bipartite.

Solution:
The resulting graph is show below.



<u>Part A</u>:

I) False. It is not possible to divide vertices 2, 3, and 4 into two groups without a connection between two vertices in the same group.

II) False. There are missing connections between several pairs of vertices (e.g., 1 and 5).

III) False. There are vertices with odd degree.

IV) True. The figure demonstrates this.

<u>Part B</u>:

V) False. For this graph, removing any vertex maintains the connectivity of the remaining vertices.

VI) True. A subgraph of a planar graph is planar.

VII) False. Even if a vertex of odd degree is removed, there are still other vertices of odd degree remaining.

VIII) False. In addition to the triangle 2, 3, 4 from Part A, there is also the triangle 5, 6, 7. It is not possible to break both with the removal of just one vertex.

Question 5:

Consider an initially empty AVL tree. The following elements are inserted in the following order: 30, 19, 42, 10, 24, 35, 51, 6, 14. After inserting all these elements, for each of the following statements about the resulting tree, indicate whether it is true or false? Justify.

I) The tree has a height of 3.

II) The tree contains exactly 4 leaves.

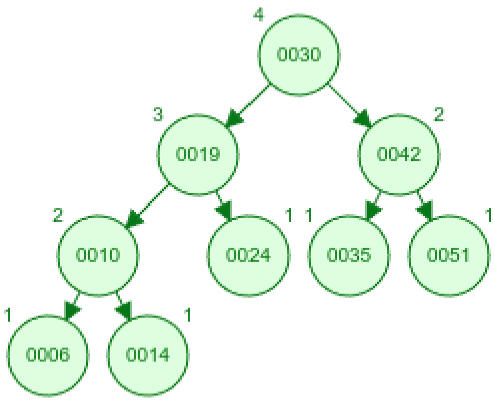
III) The node with value 30 has only one child.

IV) All nodes in the tree have balance factors ranging from -1 to 1.

V) The tree contains a node with a balance factor of 2.

Solution:

The resulting tree is show below:



I) False. The depth is 4.

II) False. There are 5 leaves.

III) False. The root has two children.

IV) True. This is the basic property of AVL trees.

V) False. If true, it would violate the AVL tree property.