University of São Paulo

São Carlos Institute of Physics

Graduate Program

Admission Test

Computational Physics

First Semester 2024

Answer Key

Candidate's Code:

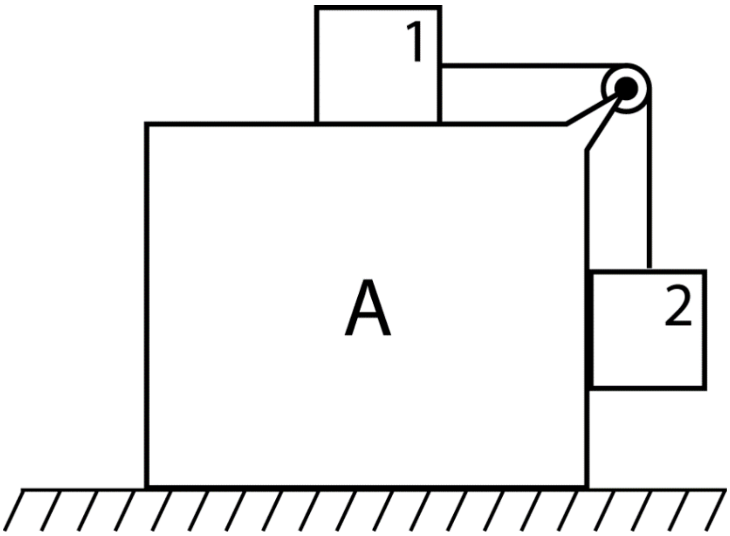
Physics Questions

(Multiple Choice)

Instructions: The Physics questions are multiple-choice. For these questions, please indicate your chosen answer directly in this exam booklet by marking the corresponding square with an "X" using a black or blue pen. Do not use this exam booklet for elaborating on your answers or as a draft. You may use the provided Notepaper for developing your answers or as a draft paper. **The answers considered for correction will be the ones marked in the exam booklet.**

Question 1:

In the given figure, the masses of bodies 1 and 2 are equal, and the coefficient of static friction between body A and bodies 1 and 2 is represented by μ. The masses of the pulley and ropes are negligible, and there is no friction in the pulleys. What is the minimum acceleration at which body A must be horizontally displaced to keep bodies 1 and 2 stationary relative to it?



Place an ‘X’ in the square that corresponds to the correct answer.

☐ $a_{min} = [(1-2\mu)/(1+\mu)]g$

☐ $a_{min} = [(1+\mu)/(1-\mu)]g$

☐ $a_{min} = [(1-\mu)/(1+2\mu)]g$

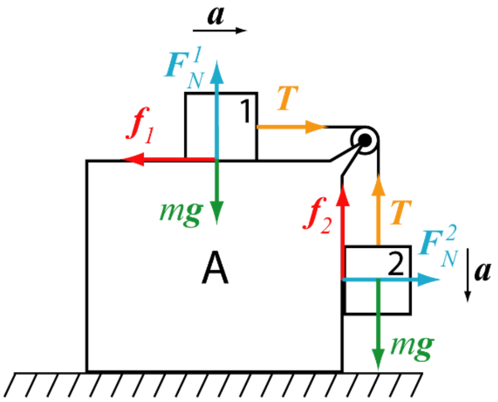
■ $a_{min} = [(1-\mu)/(1+\mu)]g$

☐ $a_{min} = [(1-2\mu)/(1+2\mu)]g$

Answer:

Bodies 1 and 2 will remain at rest with respect to body A for $a_{min} \leq a \leq a_{max}$, where a_{min} is the minimum aceleration of body A. Beyond these limits there will be relative movement between body A and the bodies 1 and 2. For $0 \leq a \leq a_{min}$ the tendency of body 1 is to move to the right in relation to body A. Based on this argument, for the purposes of calculating a_{min}, the static friction force in body 2 is directed upward and in body 1 it is directed to the left.

The force diagram for bodies 1 and 2 is then.



Let's write Newton's second law for bodies 1 and 2 in the horizontal direction.

$$T - f_1 = ma \rightarrow f_1 = T - ma \qquad (1)$$
$$F_N^2 = ma \qquad (2)$$

As body 2 has no vertical acceleration, then:

$$f_2 = mg - T \qquad (3)$$

From (1) and (3)

$$f_1 + f_2 = mg - ma \qquad (4)$$

For the situation without sliding of the bodies in relation to body A, we must have:

$$f_1 + f_2 \leq \mu(F_N^1 + F_N^2) \rightarrow f_1 + f_2 \leq \mu(mg + ma) \qquad (5)$$

Then, from (4) and (5):

$$mg - ma \leq \mu(mg + ma) \qquad (6)$$

Solving to a:

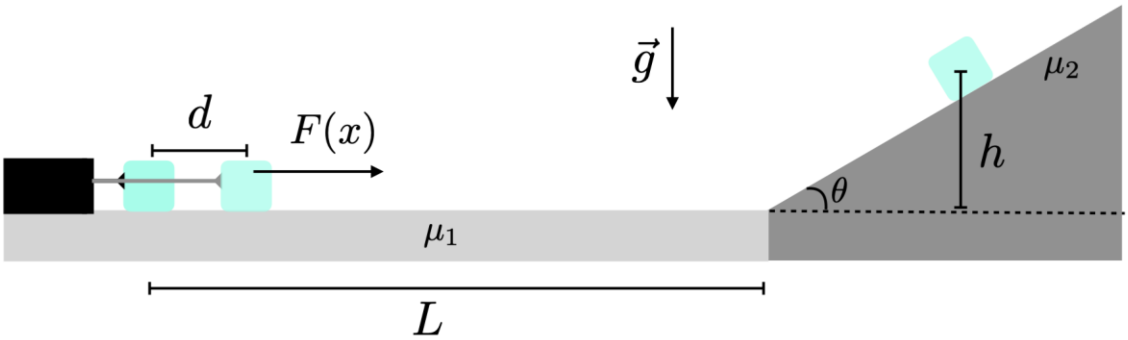
$$a \geq \frac{(1-\mu)}{(1+\mu)} g \qquad (7)$$

Therefore:

$$\boxed{a_{min} = \frac{(1-\mu)}{(1+\mu)} g}$$

Question 2:

A motor equipped with a mechanical arm pushes a block of mass $m = 1{,}0\,kg$ for a distance $d = 0{,}3\,m$, impressing on it a force $F(x) = Cx^2$ starting from the origin of the coordinate system and with the constant $C = 1000\,N/m^2$. In the first section, $L = 1{,}0\,m$ long, the coefficient of kinetic friction between the block and the surface is $\mu_1 = 0{,}1$. The block then moves up an inclined plane at an angle θ , with a coefficient of kinetic friction such that $\mu_2/tg(\theta) = 1$. Use $g = 10\,m/s^2$. The height h that the block rises before starting its descent is:



Place an 'X' in the square that corresponds to the correct answer.

☐ 0,10m

☐ 0,15m

☐ 0,20m

■ 0,40m

☐ 0,50m

Answer:

The work done by the force during the displacement d is obtained by integration, and is $Cd^3/3$. As the block slides on the plane, the force of kinetic friction does negative work against the motion, of $-\mu_1 mgL$. Then, we know the kinetic energy of the block when it starts to climb the ramp. Taking into account the work of friction in the ascent, it reaches the maximum height given by:

$$h = \frac{(Cd^3/3 - \mu_1 mgL)}{\left(mg\left(1 + \frac{\mu_2}{tg\theta}\right)\right)} = 0{,}4\ m$$

Question 3:

A simple pendulum 1 m long stays in equilibrium at an angle of $1°$ with the vertical when a fan that produces horizontal wind with speed $V = \sqrt{10}/2$ m/s is on. Here we assume that the drag force is of the type $-bV$. The fan is turned off and this pendulum then oscillates, with damping due to the drag force. Assume, throughout this question, that the oscillations are small and use appropriate approximations. Use $g = 10m/s^2$ and consider that ω_0 represents the free oscillation frequency of this pendulum.

What is the oscillation frequency of the damped pendulum for small oscillations?

Hint: for a generic oscillator with equation of motion

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \omega_0^2 x = 0$$

the oscillation frequency is $\omega = \sqrt{\omega_0^2 - (b/2m)^2}$.

Place an 'X' in the square that corresponds to the correct answer.

■ $\omega_0\sqrt{1 - \pi^2/32400}$

□ $\omega_0\sqrt{1 - \pi^2/11200}$

□ $\omega_0\sqrt{1 - \pi^2/5200}$

□ $\omega_0\sqrt{1 - \pi^2/1800}$

□ $\omega_0\sqrt{1 - \pi^2/64000}$

Answer:

First, we can find the value of b in this problem given the equilibrium condition of the pendulum. We have

$$b = \frac{mg\,\tan\theta_0}{V}$$

θ_0 is the equilibrium angle, m is the mass of the pendulum, V and g are defined in the statement.
We can write the tangential acceleration and velocity of the pendulum as a function of the angle $\theta(t)$ done with respect to vertical. We have $a = l\frac{d^2\theta}{dt^2}$ e $V = l\frac{d\theta}{dt}$, where l is the length of the pendulum. The tangential projection of the weight force is $-mg\,\text{sen}\theta$. With this we can write $F = ma$ for the pendulum taking into account the force of gravity and the drag force. The equation of motion becomes

$$\frac{d^2\theta}{dt^2} + \frac{b}{m}\frac{d\theta}{dt} + \frac{g}{l}\theta = 0$$

Where we used $\text{sen}\theta \approx \theta$. The free oscillation frequency is $\omega_0 = \sqrt{g/l}$

In the presence of the damping and using $\tan\theta \approx \theta$ t the oscillation frequency becomes,

$$\omega = \sqrt{\frac{g}{l} - \left(\frac{b}{2m}\right)^2} = \omega_0\sqrt{1 - \frac{lg\theta_0^2}{4V^2}} = \omega_0\sqrt{1 - \frac{\pi^2}{32400}}$$

Question 4:

A system consists of $0{,}32$ moles of a monatomic ideal gas, with $C_V = 3RT/2$, occupying a volume of $2{,}2\ L$ at a pressure of $2{,}4\ atm$ (point A). The system goes through a cycle that consists of three processes:

1. The gas is heated at constant pressure until its volume is $4{,}4\ L$ (point B).
2. The gas is cooled at constant volume until its pressure drops to $1{,}2\ atm$ (point C).
3. The gas undergoes an isothermal compression process until it returns to the starting point (point A).

Consider positive work as that done on the system and positive heat as that delivered to the system. What is the work, heat, and internal energy change throughout the cycle?

Place an 'X' in the square that corresponds to the correct answer.

- [ ] None of the proposed answers.
- [ ] $+\ 0{,}96\ kJ;\ -\ 0{,}96\ kJ;\quad 0{,}00\ kJ$
- [ ] $+\ 0{,}32\ kJ;\ -\ 0{,}64\ kJ;\ +\ 0{,}32\ kJ$
- [ ] $-\ 0{,}32\ kJ;\ +\ 0{,}64\ kJ;\ +\ 0{,}32\ kJ$
- [x] $-\ 0{,}16\ kJ;\ +\ 0{,}16\ kJ;\quad 0{,}00\ kJ$

Answer:

The temperatures at points A, B e C are:

$$T_C = T_A = \frac{P_A V_A}{nR} = 200\ K; \qquad T_B = \frac{P_B V_B}{nR} = \frac{P_A(2V_A)}{nR} = T_A = 400\ K;$$

The work in process 1 is: $W_1 = -P_A \Delta V = -P_A(V_B - V_A) = -535\ J$
The heat in process 1 is: $Q_1 = C_P \Delta T = \frac{5}{2} nR\Delta T = 1337\ J$
The internal energy change in process 1 is: $\Delta E_{int1} = Q_1 + W_1 = 802\ J$

The work in process 2 is: $W_2 = 0\ J$
The heat in process 2 is: $Q_2 = C_V \Delta T = \frac{3}{2} nR\Delta T = -802\ J$
Because $W_2 = 0 \rightarrow \Delta E_{int2} = Q_2 = -802\ J$

The work in process 3 is: $W_3 = nRT_A ln \frac{V_A}{V_C} = 371\ J$
The internal energy change in process 3 is: $\Delta E_{int3} = 0\ J$
The heat in process 2 is 3 é: $Q_3 = \Delta E_{int3} - W_3 = -371\ J$

Therefore:

$W_{total} = W_1 + W_2 + W_3 = -164\ J$
$Q_{total} = Q_1 + Q_2 + Q_3 = 164\ J$
$\Delta E_{int-total} = \Delta E_{int1} + \Delta E_{int2} + \Delta E_{int3} = 0\ J$

Question 5:

Consider the harmonic wave function on a string:

$$y(x,t) = (0{,}030\ m)\mathrm{sen}[(2{,}2\ m^{-1})x - (3{,}5\ s^{-1})t]$$

What is its wavelength, period, and the maximum transversal speed of any point on the string?

Place an 'X' in the square that corresponds to the correct answer.

- [ ] $0{,}4\ m;\ 1{,}7\ s;\ 1{,}1\ m/s$
- [ ] $5{,}8\ m;\ 0{,}9\ s;\ 3{,}5\ m/s$
- [x] $2{,}9\ m;\ 1{,}8\ s;\ 0{,}11\ m/s$
- [ ] $0{,}3\ m;\ 2{,}2\ s;\ 0{,}35\ m/s$
- [ ] None of the proposed answers.

Answer:

The general expression for a sinusoidal wave function is:

$$y(x,t) = A\mathrm{sen}[kx - \omega t]$$

By direct comparison we identify that for the wave function given in the problem $k = 2{,}2\ m^{-1}$ and $\omega = 3{,}5\ rad/s$.

The wavelength is $\lambda = \frac{2\pi}{k} = 2{,}9\ m$

The period is $T = \frac{2\pi}{\omega} = 1{,}8\ s$

The vertical speed is given by:

$$V_y = \frac{dy}{dt} = (-0{,}105\ m/s)\cos[(2{,}2\ m^{-1})x - (3{,}5\ s^{-1})t]$$

Therefore the maximum speed is: $V_{y,max} = 0{,}105\ m/s$

Computing Science Questions

(Open Answers)

Instructions: All questions related to computer science are in the form of open-ended questions. Therefore, you are expected to provide a comprehensive response for each question in the designated space within this examination booklet. Please use a black or blue pen for your responses. Additionally, you will receive a separate notepaper for drafting and calculations, **but only the answers provided within the allocated space in the exam booklet will be considered for evaluation.**

Question 1:

Consider the points (-1, 5), (0, 3), (1, 5), and (2, 17). Determine the interpolating polynomial that passes exactly through all of these points.

Answer:

Solution: In the general case, for a polynomial to pass through 4 points, it must have a degree of at least 3. To solve the problem, the candidate should consider the polynomial:

$ax^3 + bx^2 + cx + d$

and use the given points to set up a system of 4 equations with 4 unknowns, which are the coefficients of the polynomial:

$$a(-1)^3 + b(-1)^2 + c(-1) + d = 5$$
$$a0^3 + b0^2 + c0 + d = 3$$
$$a1^3 + b1^2 + c1 + d = 5$$
$$a2^3 + b2^2 + c2 + d = 17$$

By solving this system, you will arrive at the result, which is:

$x^3 + 2x^2 - x + 3$

Question 2:

The Pell numbers are generated by the following recurrence formula:

$$\begin{cases} p_0 = 0 \\ p_1 = 1 \\ p_n = 2p_{n-1} + p_{n-2}, n > 1 \end{cases}$$

Write a function that, given an integer *n*, returns the nth Pell number p_n.

Use C, C++, Fortran or Java in your code.

Answer:

Solution: A possible solution in C:

```
int pell(int n) {
  int p[2] = {0, 1};
  if (n < 2) return p[n];
  while (n >= 2) {
    int next = 2 * p[1] + p[0];
    p[0] = p[1];
    p[1] = next;
    --n;
  }
  return p[1];
}
```

Question 3:

Write a function that, given an array of floating-point numbers, determines the index of the element with the largest absolute value in the array, without altering the original values in the array. If there are multiple elements with the same maximum absolute value, the function should return the index of the first occurrence of such an element.

Use C, C++, Fortran or Java in your code.

Answer:

Solution: A possible solution in C:

```
size_t imax_abs(double *v, size_t N) {
  double amax = 0;
  size_t imax = 0;
  for (size_t i = 0; i < N; ++i) {
    if (fabs(v[i]) > amax) {
      amax = fabs(v[i]);
      imax = i;
    }
  }
  return imax;
}
```

Questão 4:

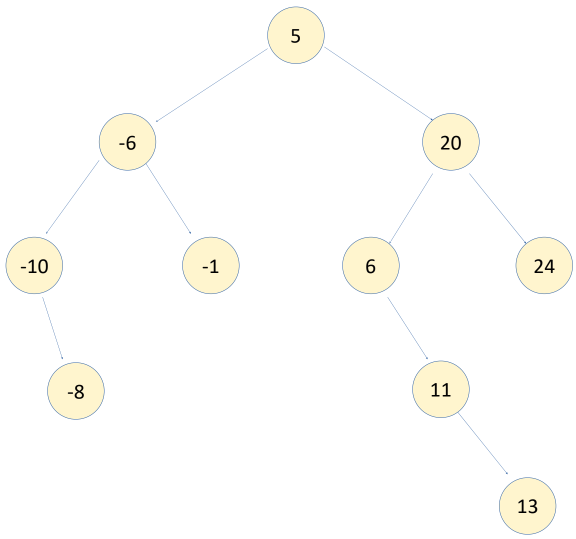
Starting with an initially empty binary search tree, insert the following values one at a time in the specified order:

5, 20, 6, -6, -10, -1, 11, 24, 13, -8

Draw the structure of the tree after all insertions.

Answer:

Solution: Figure follows:



Questão 5:

Consider an sparse, undirected and unweighted graph, where the number of edges (E) is asymptotically proportional to the number of vertices (N), meaning E = O(N). Perform a comparison between graph representations using adjacency matrix, adjacency list, and edge list in terms of their performance when executing the breadth-first search (BFS) algorithm on this type of graph.

Answer:

Solution: Given that the graph is sparsely connected, with E = O(N), the use of an adjacency matrix, which requires a representation with N^2 elements, is not an efficient choice in terms of space. Furthermore, this representation proves to be inefficient in terms of execution time during the application of the BFS algorithm. In this algorithm, it is necessary to continuously visit vertices and find their neighbors. However, searching for the neighbors of a vertex in the adjacency matrix has a cost of O(N), which would result in a BFS algorithm with a complexity of $O(N^2)$.

Both adjacency list and edge list representations are memory-efficient. However, the adjacency list is particularly well-suited for the BFS algorithm. This is because, in the adjacency list, the neighbors of each vertex are already stored directly, making neighbor search efficient. On the other hand, in the edge list, finding all neighbors of a specific vertex requires scanning the entire list of edges, making it less efficient in this context.