Nonlocal dissipative tunneling for high-fidelity quantum-state transfer between distant parties

G. D. M. Neto,[1] M. A. de Ponte,[2] and M. H. Y. Moussa[1]
[1]*Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-590 São Carlos, SP, Brazil*
[2]*Universidade Regional do Cariri, Departamento de Física, BR-63010970, Juazeiro Do Norte, CE, Brazil*
(Received 15 September 2011; published 4 May 2012)

In this work, we propose the nonlocal tunneling mechanism for high-fidelity state transfer between distant parties. The nonlocal tunneling follows from the overlap between the distant sending and receiving wave functions, which is indirectlymediated by the off-resonant normal modes of a quantum channel. This channel is made up of a network of dissipative quantum systems exhibiting the same bosonic or fermionic statistical nature as the sender and receiver. We demonstrate that the incoherence arising from quantum channel nonidealities is almost completely circumvented by the tunneling mechanism, which thus affords a high-fidelity transfer process.

DOI: 10.1103/PhysRevA.85.052303 PACS number(s): 03.67.Hk, 03.65.Ud, 03.65.Xp, 03.65.Yz

I. INTRODUCTION

Much of the recent research in the quantum information field has focused on developing new techniques to engineer long-distance entanglement [1–4] and to obtain high-fidelity communication between distant parties [5–8]. Mastery of these techniques represents a crucial step toward the practical implementation of quantum computing and communication protocols, and developments have been witnessed within both bosonic and fermionic systems. In particular, notable advances in high-efficiency state transfer have been made by adjusting the common frequency of the sender and the receiver to be in resonance with a single normal mode of the quantum channel (QC), i.e., the transmission kernel of the network [5–8]. The three-body Hamiltonian resulting from this procedure describes perfect state transfer in the ideal case in which all components of the network are lossless systems and all the relevant parameters are perfectly adjusted. Here, we take a rather different approach: the network parameters are adjusted so as to tune the degenerate sender and receiver out of resonance with all the QC normal modes. We thus observe a tunneling-like mechanism, described by a two-body Hamiltonian, which allows either a bosonic or a fermionic state to be transferred directly from the sender to the receiver, without populating the QC kernel.

Together with nonlocality [9] and the principles of superposition and collapse of quantum states [10], the tunneling mechanism [11,12] has been a central phenomenon of quantum mechanics, explored both for fundamental theoretical insights and for technological purposes. At the fundamental level, we first observe that the mechanism we are proposing here differs from the well-known tunneling effect by its nonlocal character. We then observe, from the technological standpoint, that such *nonlocal tunneling* (NLT) can be successfully employed for high-fidelity state transfer between distant systems coupled to each other through a nonideal QC. To elucidate what we call NLT, we must specify a set of attributes, starting with the fact that (i) the required overlap between the distant sender and receiver wave functions is achieved indirectly through off-resonant QC normal modes (see Fig. 1). Moreover (in analogy with the virtual excitation of an atomic transition by an off-resonant atom-field interaction), (ii) the QC is only virtually excited throughout the entire process, i.e., its constitutive systems are not populated by the initial excitation of the sender state, which is directly transferred to the receiver. Since the QC is not actually excited during the NLT, all its nonidealities are substantially weakened to the same extent as the increase in fidelity of processes such as the transfer of highly excited superposition states. Finally, (iii) during the time interval of the NLT, entanglement is developed only between the distant sender and receiver, effectively characterizing the process as nonlocal. Incidentally, the nonlocal tunneling mechanism proposed here is also suitable for preparing long-distance entanglement in noisy bosonic or fermionic networks.

We stress that in Refs. [13,14], the authors had already contemplated the regime of parameters leading to the NLT without, however, presenting an explicit derivation of a two-body Hamiltonian. Although in Ref. [14] an effective Hamiltonian is presented (for the particular case of an open antiferromagnetic Heisenberg chain), it follows from phenomenological arguments, preventing the derivation of an explicit expression for the coupling strength. In the present paper, we determine the regime of parameters which enables the explicit derivation of the two-body Hamiltonian and the associated expression for the effective coupling strength, which encompasses, as a subset, the approximations in Refs. [13,14]. Moreover, from the adiabatic elimination of the nonideal QC, we derive the expressions for the effective decay rates associated with the (assumed ideal) sender and receiver. Finally, we stress that our proposal, as that in Ref. [13], addresses networks of bosonic and fermionic systems, including coupled resonators [15,16], coupled trapped ions [17], optical lattices [18,19] and spin chains [20], assuming in this last case the application of the Jordan-Wigner fermionization technique.

After characterizing the NLT, we will then analyze the role played in it by the QC topology, i.e., the pattern in which the systems composing the QC are coupled together, their coupling strengths, and natural frequencies. To this end, we focus on a particular set of QC topologies that enable the analytical computation of a cooperativity parameter. This parameter balances the cost of the dissipation rate against the benefit of the effective sender-receiver coupling strength. Moreover, we relate NLT to earlier results on dissipative quantum tunneling through local [11] and nonlocal (two-point) [12] double-well potentials.

 ©2012 American Physical Society

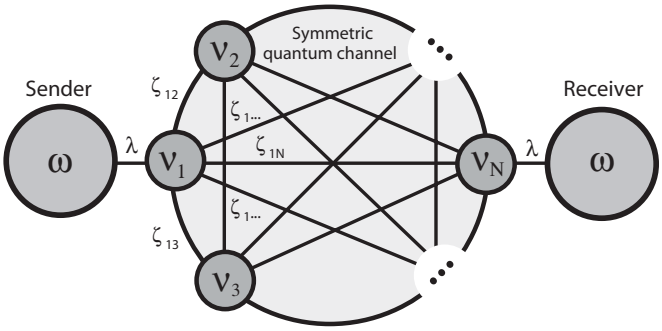

FIG. 1. A network for state transfer through nonlocal tunneling assuming a symmetric QC.

II. THE MASTER EQUATION

We start by defining the Hamiltonian $H = H_{\rm QC} + H_{S,R} + H_{\rm loss}$ describing the whole system, where the QC, whose components are characterized by the frequencies ν_m and labeled by $m,n = 1,\ldots,N$, is modeled by

$$H_{\rm QC} = \sum_{m,n} O_m^\dagger \mathcal{H}_{mn} O_n,$$

with elements $\mathcal{H}_{mn} = \nu_m \delta_{mn} + \zeta_{mn}(1-\delta_{mn})$, O_m ($O_m^\dagger$) being the annihilation (creation) bosonic or fermionic operator. We have assumed, as sketched in Fig. 1, the general perspective of a symmetric QC, where each system (m) interacts with any other (n) with coupling strength ζ_{mn}, since any particular topology follows from this one by an appropriate choice of the Hamiltonian parameters [16]. Moreover, each system composing the QC is assumed to interact, with coupling strength V_{mk}, with its own amplitude-damping reservoir, modeled by an infinite set of oscillators, as described by the Hamiltonian

$$H_{\rm loss} = \sum_{m,k} [\omega_{mk} c_{mk}^\dagger c_{mk} + V_{mk}(O_m c_{mk}^\dagger + \text{H.c.})],$$

where the kth mode ω_{mk} of the mth reservoir is described by the annihilation (creation) operator c_{mk} ($c_{mk}^\dagger$). Differently from the nonideal components of the QC, the sender (S) and the receiver (R) are assumed to be ideal systems, inside controlled (cryogenic) environments, and thus described by the Hamiltonian

$$H_{S,R} = \omega \sum_{\ell=S,R} \Lambda_\ell^\dagger \Lambda_\ell + \lambda(\Lambda_S^\dagger O_1 + \Lambda_R^\dagger O_N + \text{H.c.}),$$

Λ_ℓ ($\Lambda_\ell^\dagger$) being the annihilation (creation) bosonic or fermionic operator of both systems, S and R, characterized by the frequency ω and coupled to the first and the Nth QC components, respectively, with strength λ (see Fig. 1). To derive the master equation for the network density operator ρ, we first diagonalize $H_{\rm QC}$ by applying the transformation $R_m = \sum_n T_{mn}^{-1} O_n$, where the coefficients of the nth column of the orthonormal matrix $\mathbf{T}$ ($\mathbf{T}^{-1} = \mathbf{T}^{\rm T}$) define the eigenvectors associated with the eigenvalues $\bar{\nu}_m$ [16]. After tracing out the degrees of freedom of the reservoirs, we obtain

$$\dot{\rho} = -i[\tilde{H},\rho] + \sum_m (\gamma_m/2)([R_m\rho, R_m^\dagger] + [R_m, \rho R_m^\dagger]), \quad (1)$$

where γ_m stands for the damping rate of the mth normal mode, and the Hamiltonian $\tilde{H} = \tilde{H}_{\rm QC} + \tilde{H}_{S,R}$ includes the terms $\tilde{H}_{\rm QC} = \sum_m \bar{\nu}_m R_m^\dagger R_m$ and $\tilde{H}_{S,R} = \omega \sum_\ell \Lambda_\ell^\dagger \Lambda_\ell + \lambda \sum_m [(T_{1m}\Lambda_S^\dagger + T_{Nm}\Lambda_R^\dagger) R_m + \text{H.c.}]$.

III. EFFECTIVE COUPLING BETWEEN THE SENDER AND THE RECEIVER

In order to engineer an effective sender-receiver interaction, we first set these systems (ω) to be dispersively coupled to all the QC normal modes ($\bar{\nu}_m$), such that the condition $\sqrt{\bar{n}}\lambda/\Delta_m \ll 1$ is satisfied, $\bar{n}$ being the mean excitation of the sender initial state and $\Delta_m = |\bar{\nu}_m - \omega|$. Moreover, the coupling strength λ of the sender and the receiver with the QC modes must be much smaller than those between the QC systems ζ_{mn}, i.e., $\lambda \ll \zeta_{mn}$. Otherwise, the excitation of the state to be transferred would populate the QC, even under a large detuning Δ_m. The above restrictions allow us to eliminate the QC variables adiabatically [21], to obtain, up to second order in λ, the equation for the reduced "sender+receiver" density operator $\rho_{S,R}$:

$$\dot{\rho}_{S,R} = -i[H_{\rm eff}, \rho_{S,R}] + \sum_\ell (\Gamma_\ell/2)([\Lambda_\ell \rho_{S,R}, \Lambda_\ell^\dagger] + [\Lambda_\ell, \rho_{S,R}\Lambda_\ell^\dagger]), \quad (2)$$

where

$$H_{\rm eff} = \chi(\Lambda_S^\dagger \Lambda_R + \Lambda_S \Lambda_R^\dagger) \quad (3)$$

describes the effective sender-receiver coupling, with strength $\chi = \sum_m \lambda^2 |T_{m1} T_{mN}|/\Delta_m$ [22]. We must also observe that $H_{\rm eff}$ is restricted to networks where $N \ll \Delta_m/\sqrt{\bar{n}}\lambda$, since its third-order correction is proportional to the factor $N\lambda^3/\Delta_m^2$, and this has to be much smaller than λ^2/Δ_m. Moreover, the simplified Lindblad form of the superoperator in Eq. (2) follows from the condition that the QC constituents are initialized in their ground state. Otherwise, with an initially excited QC, we end up with a different reduced master equation (2) and an effective sender-receiver interaction distinct from the Josephson-like structure in Eq. (3). Finally, we point out that the dissipative mechanisms of all the QC systems are taken into account through the effective damping rates associated with the sender and receiver, given by

$$\Gamma_\ell = \sum_m \gamma_m (T_{m1}\delta_{\ell S} + T_{mN}\delta_{\ell R})^2 \lambda^2/\Delta_m^2, \quad (4)$$

which, like χ, depend on the QC topology. The regime of parameters that validates the effective Hamiltonian (3) leads to vanishing cross-decay terms Γ_{SR} and Γ_{RS} [16].

From here on we assume the particular case of equal natural frequencies ν and damping rates γ for all the QC normal modes. We also assume equal coupling strengths ζ between the QC systems, in the regime where $\zeta \ll \nu$, adopted to enable analytical results. In this regime, the condition $\sqrt{\bar{n}}\lambda/\Delta_m \ll 1$ is satisfied by all m normal-mode frequencies. In fact, the small couplings between the QC systems induce an equally small bandwidth of the normal-mode frequencies, which fall within the interval $[\nu - N\zeta, \nu + N\zeta]$, thus satisfying $\sqrt{\bar{n}}\lambda/\Delta \ll 1$, all Δ_m having approximately the same value $\Delta = |\nu - \omega|$. Under the parameters defined above, we thus

obtain from Eq. (4) the attenuated effective decay rates, $\Gamma_\ell \sim \gamma\,(\lambda/\Delta)^2 \ll \gamma$, as well as the excitation of the QC: $\mathrm{Tr}_{\mathrm{QC}}(\rho_{\mathrm{QC}} \sum_m R_m^\dagger R_m) \approx \bar{n}(\lambda/\Delta)^2 e^{-\gamma t} \ll \bar{n}$, which confirms that it is only virtually excited under the approximation for H_{eff}.

IV. COOPERATIVITY PARAMETER AND TRANSFER TIME

To establish a metric to identify the optimal topology of the QC—within the specific set of topologies leading to the effective interaction H_{eff} and the coupling regime $\zeta \ll \nu$—we turn to the cooperativity parameter $\mathcal{C}$ and the transfer time. Assuming, for simplicity, $\Gamma_S = \Gamma_R = \Gamma$, the cooperativity

$$\mathcal{C} = \frac{\Gamma}{\chi} = \frac{\gamma \sum_m \left(T_{m1}^2 + T_{mN}^2\right)}{\Delta \sum_m T_{m1} T_{mN}}$$

weighs the cost of the dissipation rate against the benefit of the coupling strength. To compute the time interval needed to accomplish the nonlocal tunneling, we start from the solution $\rho_{S,R}$ of Eq. (2). After tracing out the degrees of freedom of the sender, we get ρ_R, from which we compute the fidelity for the state transfer: $\mathcal{F}_R(t) = \mathrm{Tr}_R\,[\rho_R(t)\,|\psi_R(0)\rangle\,\langle\psi_R(0)|]$, with $|\psi_R(0)\rangle \equiv |\psi_S(0)\rangle$. The maximization of $\mathcal{F}_R(t)$ leads to the expected transfer time, $\tau = (1/\eta)\cot^{-1}(\Gamma/4\eta) \approx \pi/2\eta$, where $\eta = \chi\sqrt{1-(\mathcal{C}/4)^2}$.

V. NETWORK TOPOLOGIES

To encompass a large number of QC topologies enabling analytical results for $\mathcal{C}$, we restrict our analysis to a specific set of topologies whose components have an increasing number of connections between them, associated with first-, second-, ..., and Jth-neighbor couplings, with J running from 1 to int$(N/2)$. The case $J = 1$ gives a circular QC, where the kth resonator is coupled to the $(k \pm 1)$th, while the case $J = \mathrm{int}(N/2)$ leads to the symmetric network in Fig. 1(a). The normal-mode frequencies of all the networks—the circular, the symmetric, and all the $[\mathrm{int}(N/2)+1]/2$ intermediate topologies—are given by $\bar{\nu}_m = \nu + \zeta \sum_{k=1}^{J} \left(2 - \delta_{2k,N}\right) \cos\left(2mk\pi/N\right)$. For the cases in which $2m/N = 0,1$, we find that the eigenvalues $\bar{\nu}_m$ are not degenerate, the coefficients of their associated eigenvectors being $T_{mn} = \mathcal{N}\cos\left[2\pi m\,(n-1)/N\right]$, where $\mathcal{N} = \sqrt{2/(N+1)}$. On the other hand, when $2m/N \neq 0,1$, the eigenvalues are twofold-degenerate, the coefficients of both linearly independent eigenvectors being the above sinusoidal T_{mn} and $\tilde{T}_{mn} = \mathcal{N}\sin\left[2\pi m\,(n-1)/N\right]$. From the equations defining the eigenvalues and eigenvectors arising from the diagonalized $\tilde{H}_{\mathrm{QC}}$, we compute analytically the cooperativity parameters

$$\mathcal{C}_J = J\gamma/\Delta, \tag{5}$$

implying that the fewer connections are made between the systems, the higher the fidelity of the transferred state. Regardingthe transfer time $\tau \approx \pi/2\eta$, after computing the coupling strength $\chi = \lambda^2/2J\Delta$, we obtain the relation $\tau_J = \pi J\Delta/\left[\lambda^2 - (J\gamma\lambda/4\Delta)^2\right]$, which under the reasonable approximation $J\gamma \ll \gamma\Delta/\lambda \ll \Delta$—adopted from here on and holding when the decay factor is significantly smaller than the detuning Δ—simplifies to

$$\tau_J = \pi/2\chi = \pi J\Delta/\lambda^2. \tag{6}$$

Therefore, while it is independent of the state to be transferred, the transfer time can be controlled through the number of connections J, being shortest in the linear (or circular) topology.

Focusing then on the linear topology, we next consider a bosonic network to analyze the transfer of an even (+) or odd (−) Schrödinger-cat-like state $|\psi_\pm(0)\rangle = \mathcal{N}_\pm\left(|\alpha\rangle \pm |-\alpha\rangle\right)$ and compute, at the transfer time $\tau = \pi\Delta/\lambda^2$, the fidelity $\mathcal{F}_\pm(\tau) \approx (1 + e^{-\pi|\alpha|^2\mathcal{C}/2})/2$, which is maximized by minimizing the cooperativity $\mathcal{C}$. Considering an array of toroidal or spherical microcavities coupled by optical fibers [15], the condition for dispersive coupling between the "sender + receiver" system and the QC can be fulfilled by first setting N and then adjusting the gap between the fibers and the microcavities, to engineer the coupling strength $\lambda \ll \Delta/N$. For ω and ν in the microwave region, with $\Delta \approx 10^9$ s^{-1} and $\lambda \approx 10^6$ s^{-1}, we obtain the ratio $\lambda/\Delta \approx 10^{-3}$, which enables us to achieve a quasiperfect state transfer within a network of around 100 cavities. With the quality factor Q of these cavities around 10^5, such that $\gamma = \omega/Q \approx 10^5$, we obtain, for $J = 1$, the cooperativity parameter $\mathcal{C} \approx 10^{-4}$, the transfer time $\tau \approx 10^{-3}$ s, and the fidelity $\mathcal{F}_\pm$ around unity, as long as $|\alpha|^2\,\mathcal{C} \ll 1$, i.e., for coherent states with mean excitations up to around 10^3. Still regarding a linear QC, in Fig. 2 the dashed (dotted) line indicates the falling (rising) excitation in the sender (receiver) cavity, scaled by the excitation $|\alpha|^2$ of the sender initial state. The dashed-dotted line, the unit sum of both these scaled excitations, given by $\sum_\ell \mathrm{Tr}_\ell(\rho_\ell a_\ell^\dagger a_\ell)/\,|\alpha|^2$, indicates that the QC is effectively unpopulated. The solid line displays the concurrence, on the same unit scale, of the gradual entanglement between the distant sender and receiver resonators, which characterizes the tunneling as a nonlocal phenomenon. The maximum degree of entanglement is attained at exactly half the transfer time. A similar analysis can be carried out for the tunneling of massive particles in

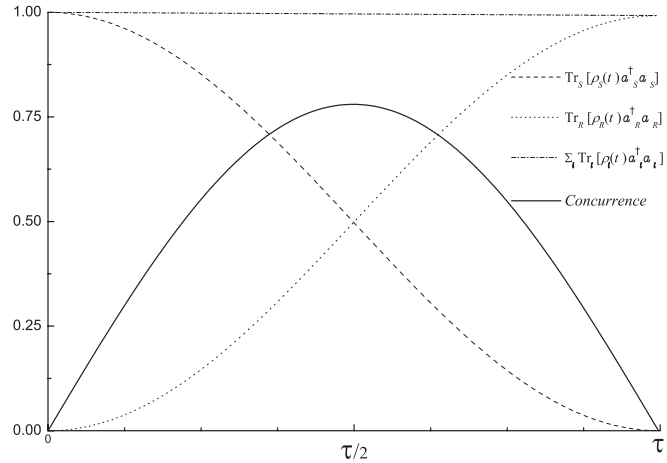

FIG. 2. Time evolution of the excitations in the sender (dashed line) and the receiver (dotted line) resonators, scaled by $|\alpha|^2$. The sum of both these scaled excitations is indicated by the dashed-dotted line, whereas the solid line displays the concurrence of the entanglement between the two resonators.

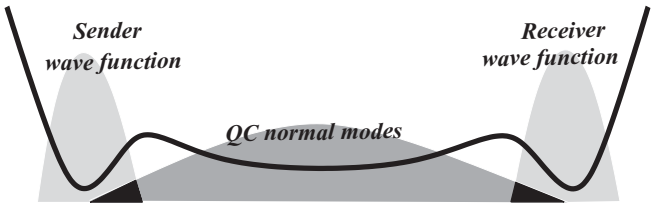

FIG. 3. Sketch of the effective potential configuration to illustrate the indirect overlap between the sender and the receiver wave functions through the QC.

optical lattices where, assuming the confinement of the atoms on each lattice site to be around $\omega \approx 10^4$ s^{-1} [19], such that $\Delta \approx 10^4$ s^{-1}, and the tunnel coupling between neighboring lattice sites around $\lambda \approx 10^3$ s^{-1}, we obtain the transfer time around $\tau \approx 10^{-1}$ s.

VI. DISSIPATIVE QUANTUM TUNNELING AND NONLOCALITY

Next, we analyze the NLT in terms of the well-known tunneling effect in a double-well barrier, where the sender and the receiver play the part of two wells separated by a potential barrier represented by the QC. We first address the ideal nondissipative case with $J = 1$, where the transfer time (6) can be estimated, as in the literature [12], from the energy gap ΔE associated with the effective coupling H_{eff}: $\tau = \pi/\Delta E$. In fact, from H_{eff} we obtain the normal-mode energies $\omega \pm \chi$, leading to $\Delta E = 2\chi = \lambda^2/\Delta$ and, consequently, to Eq. (6). Regarding dissipative quantum tunneling, we observe that our computed excitation inversion $\mathcal{P}(t) = \sum_\ell (-1)^{\delta_{\ell A}} \text{Tr}_\ell[\rho_\ell(t) a_\ell^\dagger a_\ell] = [\cos(2\eta t) - (\mathcal{C}/4)\sin(2\eta t)]\, e^{-\Gamma t}$ is closely related to the corresponding expression obtained by Leggett *et al.* [11] when studying the dynamics of a two-state system coupled to a dissipative environment. In Ref. [11], the authors derive the same behavior for $\mathcal{P}(t)$ when analyzing a super-Ohmic spectral density at $T = 0$ K and $\mathcal{C} \ll 1$ (exactly as in our analysis), the only case leading to underdamped coherent oscillations, which bears a striking resemblance to our NLT. We must emphasize, however, that our tunneling mechanism, different from those in the literature, is a *nonlocal* process in that the sender and the receiver can be far apart from each other, the coupling between their wave functions being *indirectly* mediated by the QC normal modes. Pursuing further our attempt to draw an analogy with an effective potential configuration, our NLT network can be seen, as in Fig. 3, as a double-well barrier with the wells far apart (trapping the sender and the receiver wave functions) and separated by a potential represented by another wide and shallow well at a higher energy level (when $\nu - \omega > 0$), holding the closely spaced QC normal modes. Again, the tunneling interaction H_{eff} is indirectly accomplished by the overlaps, indicated by darker regions in Fig. 3, between the sender and the receiver wave functions and the QC normal modes.

A. (Two-point) nonlocal potentials

Regarding the influence of the number of connections J on the tunneling time (6), it displays a close resemblance to the effects of nonlocal potentials (in the sense of being two-point functions) in double-well barriers. It is known that, apart from increasing with J, as we have found, the tunneling time also grows proportionally to the degree of nonlocality [12]. In fact, we may expect an increasing J to simulate the strengthening of the nonlocal two-point potential. To be more precise, the underlying mechanism of this strengthening is the weight each normal mode contributes to the effective coupling $\chi = \lambda^2 |T_{m1} T_{mN}|/\Delta_m$. As can be verified from the expression derived above for $\bar{\nu}_m$, as J increases, so does the number of the QC normal modes away from the sender and the receiver frequency ω, thus increasing the detuning Δ_m and so decreasing $\chi (\propto \tau^{-1})$.

VII. CONCLUDING REMARKS

By considering two distant quantum systems—the sender and the receiver—coupled to each other through a nonideal QC—a network composed of either fermionic or bosonic systems—we have succeeded in deriving a two-body Hamiltonian describing what we have characterized as nonlocal dissipative tunneling. We have demonstrated the development of an entanglement between the sender and the receiver, of which the QC does not take part, that definitely characterizes the tunneling as a nonlocal mechanism. This mechanism thus becomes well-suited for high-fidelity quantum state transfer between distant parties since the QC is only virtually excited throughout the entire process, causing its nonidealities to be substantially weakened. We remark that, beyond the numerical analysis developed in Refs. [13,14] to characterize the NLT, we present here the formal derivation of the effective Hamiltonian under a stringent regime of parameters.

As already mentioned above, apart from high-fidelity state transfer, the NLT is also suitable for preparing long-distance entanglement in noisy bosonic or fermionic networks. We hope that the derivations presented here may be useful for other purposes rather than those in quantum information theory.

ACKNOWLEDGMENTS

The authors acknowledge financial support from FAPESP, CNPQ, and CAPES, Brazilian agencies.

[1] L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia, Phys. Rev. Lett. **96**, 247206 (2006).
[2] S. Broadfoot, U. Dorner, and D. Jaksch, Phys. Rev. A **82**, 042326 (2010).
[3] J. B. Brask, I. Rigas, E. S. Polzik, U. L. Andersen, and A. S. Sorensen, Phys. Rev. Lett. **105**, 160501 (2010).
[4] S. M. Giampaolo and F. Illuminati, New J. Phys. **12**, 025019 (2010).

[5] M. B. Plenio and F. L. Semiao, New J. Phys. **7**, 73 (2005).
[6] L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia, Phys. Rev. Lett. **99**, 060401 (2007).
[7] A. Cacheffo, M. A. de Ponte, M. H. Y. Moussa, and A. S. M. de Castro, J. Phys. B **43**, 105503 (2010).
[8] N. Y. Yao, L. Jiang, A. V. Gorshkov, Z. X. Gong, A. Zhai, L. M. Duan, and M. D. Lukin, Phys. Rev. Lett. **106**, 040505 (2011).
[9] A. Einstein *et al.*, Phys. Rev. **47**, 777 (1935); J. S. Bell, Ann. Phys. (NY) **1**, 195 (1964); P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995).
[10] P. A. M. Dirac, *Quantum Mechanics* (Oxford University Press, London, 1958).
[11] A. J. Leggett *et al.*, Rev. Mod. Phys. **59**, 1 (1987).
[12] M. Razavy, *Quantum Theory of Tunneling* (World Scientific, Singapore, 2003).
[13] S. Paganelli, F. de Pasquale, and G. L. Giorgi, Phys. Rev. A **74**, 012316 (2006).
[14] L. Campos Venuti, S. M. Giampaolo, F. Illuminati, and P. Zanardi, Phys. Rev. A **76**, 052328 (2007).
[15] K. J. Vahala, Nature (London) **424**, 839 (2003).
[16] M. A. dePonte, S. S. Mizrahi, and M. H. Y. Moussa, Phys. Rev. A **76**, 032101 (2007).
[17] M. Johanning, J. Phys. B **42**, 154009 (2009).
[18] M. Lewenstein *et al.*, Adv. Phys. **56**, 243 (2007).
[19] I. Bloch, Nat. Phys. **1**, 23 (2005).
[20] A. Kay, Int. J. Quantum. Inform. **8**, 641 (2010).
[21] A. B. Klimov and L. L. Sánchez-Soto, Phys. Rev. A **61**, 063802 (2000); I. Sainz, A. B. Klimov, and L. Roa, *ibid.* **73**, 032303 (2006).
[22] The effective Hamiltonian (3), without taking into account the nonidealities of the QC, may be derived in a simpler form through the approximation technique in D. F. V. James and J. Jerke, Can. J. Phys. **85**, 625 (2007).